GCSE Mathematics (1MA1)
Higher Tier

Scheme of Work

Higher tier

Unit		Title	Estimated hours
1	a	Calculations, checking and rounding	4
	b	Indices, roots, reciprocals and hierarchy of operations	4
	c	Factors, multiples, primes, standard form and surds	7
2	a	Algebra: the basics, setting up, rearranging and solving equations	10
	b	Sequences	4
3	a	Averages and range	4
	b	Representing and interpreting data and scatter graphs	5
4	a	Fractions and percentages	12
	b	Ratio and proportion	6
5	a	Polygons, angles and parallel lines	6
	b	Pythagoras' Theorem and trigonometry	6
6	a	Graphs: the basics and real-life graphs	6
	b	Linear graphs and coordinate geometry	8
	c	Quadratic, cubic and other graphs	6
7	a	Perimeter, area and circles	5
	b	3D forms and volume, cylinders, cones and spheres	7
	c	Accuracy and bounds	5
8	a	Transformations	6
	b	Constructions, loci and bearings	7
9	a	Solving quadratic and simultaneous equations	7
	b	Inequalities	6
10		Probability	8
11		Multiplicative reasoning	8
12		Similarity and congruence in 2D and 3D	6
13	a	Graphs of trigonometric functions	6
	b	Further trigonometry	9
14	a	Collecting data	4
	b	Cumulative frequency, box plots and histograms	6
15		Quadratics, expanding more than two brackets, sketching graphs, graphs of circles, cubes and quadratics	7
16	a	Circle theorems	5
	b	Circle geometry	5
17		Changing the subject of formulae (more complex), algebraic fractions, solving equations arising from algebraic fractions, rationalising surds, proof	7
18		Vectors and geometric proof	9
19	a	Reciprocal and exponential graphs; Gradient and area under graphs	7
	b	Direct and inverse proportion	7

UNIT 1: Powers, decimals, HCF and LCM, positive and negative, roots, rounding, reciprocals, standard form, indices and surds

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SPECIFICATION REFERENCES

N2 apply the four operations, including formal written methods, to integers, decimals … both positive and negative; understand and use place value (e.g. working with very large or very small numbers, and when calculating with decimals)

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

N4 use the concepts and vocabulary of prime numbers, factors (divisors), multiples, common factors, common multiples, highest common factor, lowest common multiple, prime factorisation, including using product notation and the unique factorisation theorem

N5 apply systematic listing strategies **including use of the product rule for counting (i.e. if there are m ways of doing one task and for each of these, there are n ways of doing another task, then the total number of ways the two tasks can be done is $m \times n$ ways)**

N6 use positive integer powers and associated real roots (square, cube and higher), recognise powers of 2, 3, 4, 5; **estimate powers and roots of any given positive number**

N7 calculate with roots and with integer **and fractional** indices

N8 calculate exactly with … **surds**; … **simplify surd expressions involving squares (e.g. $\sqrt{12} = \sqrt{(4 \times 3)} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$)**

N9 calculate with and interpret standard form $A \times 10^n$, where $1 \le A < 10$ and n is an integer.

N14 estimate answers; check calculations using approximation and estimation, including answers obtained using technology

N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); …

PRIOR KNOWLEDGE

It is essential that students have a firm grasp of place value and be able to order integers and decimals and use the four operations.
Students should have knowledge of integer complements to 10 and to 100, multiplication facts to 10 × 10, strategies for multiplying and dividing by 10, 100 and 1000.
Students will have encountered squares, square roots, cubes and cube roots and have knowledge of classifying integers.

KEYWORDS

Integer, number, digit, negative, decimal, addition, subtraction, multiplication, division, remainder, operation, estimate, power, roots, factor, multiple, primes, square, cube, even, odd, surd, rational, irrational standard form, simplify

1a. Calculations, checking and rounding (N2, N3, N5, N14, N15)	**Teaching time** 3-5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Add, subtract, multiply and divide decimals, whole numbers including any number between 0 and 1;
- Put digits in the correct place in a decimal calculation and use one calculation to find the answer to another;
- Use the product rule for counting (i.e. if there are m ways of doing one task and for each of these, there are n ways of doing another task, then the total number of ways the two tasks can be done is $m \times n$ ways);
- Round numbers to the nearest 10, 100, 1000, the nearest integer, to a given number of decimal places and to a given number of significant figures;
- Estimate answers to one- or two-step calculations, including use of rounding numbers and formal estimation to 1 significant figure: mainly whole numbers and then decimals.

POSSIBLE SUCCESS CRITERIA

Given 5 digits, what is the largest even number, largest odd number, or largest or smallest answers when subtracting a two-digit number from a three-digit number?
Given 2.6 × 15.8 = 41.08 what is 26 × 0.158? What is 4108 ÷ 26?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that include providing reasons as to whether an answer is an overestimate or underestimate.
Missing digits in calculations involving the four operations.
Questions such as: Phil states 3.44 × 10 = 34.4, and Chris states 3.44 × 10 = 34.40. Who is correct?
Show me another number with 3, 4, 5, 6, 7 digits that includes a 6 with the same value as the "6" in the following number 36 754.

COMMON MISCONCEPTIONS

Significant figure and decimal place rounding are often confused.
Some pupils may think 35 934 = 36 to two significant figures.

NOTES

The expectation for Higher tier is that much of this work will be reinforced throughout the course.
Particular emphasis should be given to the importance of clear presentation of work.
Formal written methods of addition, subtraction and multiplication work from right to left, whilst formal division works from left to right.
Any correct method of multiplication will still gain full marks, for example, the grid method, the traditional method, Napier's bones.
Encourage the exploration of different calculation methods.
Amounts of money should always be rounded to the nearest penny.
Make sure students are absolutely clear about the difference between significant figures and decimal places.

1b. Indices, roots, reciprocals and hierarchy of operations (N2, N3, N6, N7, N14)	**Teaching time** 3-5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use index notation for integer powers of 10, including negative powers;
- Recognise powers of 2, 3, 4, 5;
- Use the square, cube and power keys on a calculator and estimate powers and roots of any given positive number, by considering the values it must lie between, e.g. the square root of 42 must be between 6 and 7;
- Find the value of calculations using indices including positive, fractional and negative indices;
- Recall that $n^0 = 1$ and $n^{-1} = \frac{1}{n}$ for positive integers n as well as, $n^{\frac{1}{2}} = \sqrt{n}$ and $n^{\frac{1}{3}} = \sqrt[3]{n}$ for any positive number n;
- Understand that the inverse operation of raising a positive number to a power n is raising the result of this operation to the power $\frac{1}{n}$;
- Use index laws to simplify and calculate the value of numerical expressions involving multiplication and division of integer powers, fractional and negative powers, and powers of a power;
- Solve problems using index laws;
- Use brackets and the hierarchy of operations up to and including with powers and roots inside the brackets, or raising brackets to powers or taking roots of brackets;
- Use an extended range of calculator functions, including +, −, ×, ÷, x^2, $\sqrt{x}$, memory, x^y, $x^{\frac{1}{y}}$, brackets;
- Use calculators for all calculations: positive and negative numbers, brackets, powers and roots, four operations.

POSSIBLE SUCCESS CRITERIA

What is the value of 2^5?
Prove that the square root of 45 lies between 6 and 7.

Evaluate $(2^3 \times 2^5) \div 2^4$, 4^0, $8^{-\frac{2}{3}}$.
Work out the value of n in $40 = 5 \times 2^n$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that use indices instead of integers will provide rich opportunities to apply the knowledge in this unit in other areas of Mathematics.

COMMON MISCONCEPTIONS

The order of operations is often not applied correctly when squaring negative numbers, and many calculators will reinforce this misconception.

NOTES

Students need to know how to enter negative numbers into their calculator.
Use negative number and not minus number to avoid confusion with calculations.

1c. Factors, multiples, primes, standard form and surds (N3, N4, N8, N9)	**Teaching time** 6-8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Identify factors, multiples and prime numbers;
- Find the prime factor decomposition of positive integers – write as a product using index notation;
- Find common factors and common multiples of two numbers;
- Find the LCM and HCF of two numbers, by listing, Venn diagrams and using prime factors – include finding LCM and HCF given the prime factorisation of two numbers;
- Solve problems using HCF and LCM, and prime numbers;
- Understand that the prime factor decomposition of a positive integer is unique, whichever factor pair you start with, and that every number can be written as a product of prime factors;
- Convert large and small numbers into standard form and vice versa;
- Add, subtract, multiply and divide numbers in standard form;
- Interpret a calculator display using standard form and know how to enter numbers in standard form;
- Understand surd notation, e.g. calculator gives answer to sq rt 8 as 4 rt 2;
- Simplify surd expressions involving squares (e.g. $\sqrt{12} = \sqrt{(4 \times 3)} = \sqrt{4} \times \sqrt{3} = 2\sqrt{3}$).

POSSIBLE SUCCESS CRITERIA

Know how to test if a number up to 120 is prime.
Understand that every number can be written as a unique product of its prime factors.
Recall prime numbers up to 100.
Understand the meaning of prime factor.
Write a number as a product of its prime factors.
Use a Venn diagram to sort information.
Write 51080 in standard form.
Write 3.74×10^{-6} as an ordinary number.
Simplify $\sqrt{8}$.
Convert a 'near miss', or any number, into standard form; e.g. 23×10^7.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements and justify which answer is correct by providing a counter-argument by way of a correct solution.
Links with other areas of Mathematics can be made by using surds in Pythagoras and when using trigonometric ratios.

COMMON MISCONCEPTIONS

1 is a prime number.
Particular emphasis should be made on the definition of "product" as multiplication, as many students get confused and think it relates to addition.
Some students may think that any number multiplied by a power of ten qualifies as a number written in standard form.
When rounding to significant figures some students may think, for example, that 6729 rounded to one significant figure is 7.

NOTES

Use a number square to find primes (Eratosthenes sieve).
Using a calculator to check the factors of large numbers can be useful.
Students need to be encouraged to learn squares from 2×2 to 15×15 and cubes of 2, 3, 4, 5 and 10, and corresponding square and cube roots.
Standard form is used in science and there are lots of cross-curricular opportunities.
Students need to be provided with plenty of practice in using standard form with calculators.
Rationalising the denominator is covered later in unit 17.

UNIT 2: Expressions, substituting into simple formulae, expanding and factorising, equations, sequences and inequalities, simple proof

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SPECIFICATION REFERENCES

N1 … use the symbols =, ≠, <, >, ≤, ≥

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals

N8 calculate exactly with fractions, **surds** …; **simplify surd expressions involving squares** …

N9 calculate with and interpret standard form $A \times 10^n$, where $1 \leq A < 10$ and n is an integer.

A1 use and interpret algebraic notation, including:

- ab in place of $a \times b$
- $3y$ in place of $y + y + y$ and $3 \times y$
- a^2 in place of $a \times a$, a^3 in place of $a \times a \times a$, a^2b in place of $a \times a \times b$
- $\frac{a}{b}$ in place of $a \div b$
- coefficients written as fractions rather than as decimals
- brackets

A2 substitute numerical values into formulae and expressions, including scientific formulae

A3 understand and use the concepts and vocabulary of expressions, equations, formulae, identities, inequalities, terms and factors

A4 simplify and manipulate algebraic expressions … by:

- collecting like terms
- multiplying a single term over a bracket
- taking out common factors
- expanding products of two … binomials
- factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares; …
- simplifying expressions involving sums, products and powers, including the laws of indices

A5 understand and use standard mathematical formulae; rearrange formulae to change the subject

A6 know the difference between an equation and an identity; argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **and proofs**

A7 where appropriate, interpret simple expressions as functions with inputs and outputs; …

A17 solve linear equations in one unknown algebraically …;

A20 **find approximate solutions to equations numerically using iteration**

A21 translate simple situations or procedures into algebraic expressions or formulae; derive an equation …, solve the equation and interpret the solution

A23 generate terms of a sequence from either a term-to-term or a position-to-term rule

A24 recognise and use sequences of triangular, square and cube numbers, simple arithmetic progressions, Fibonacci type sequences and simple geometric progressions (r^n where n is an integer, and r is a rational number > 0), recognise and use other sequences **or a surd)**

A25 deduce expressions to calculate the nth term of linear sequences.

PRIOR KNOWLEDGE

Students should have prior knowledge of some of these topics, as they are encountered at Key Stage 3:

- the ability to use negative numbers with the four operations and recall and use hierarchy of operations and understand inverse operations;
- dealing with decimals and negatives on a calculator;
- using index laws numerically.

KEYWORDS

Expression, identity, equation, formula, substitute, term, 'like' terms, index, power, negative and fractional indices, collect, substitute, expand, bracket, factor, factorise, quadratic, linear, simplify, approximate, arithmetic, geometric, function, sequence, *n*th term, derive

2a. Algebra: the basics, setting up, rearranging and solving equations	**Teaching time** 9-11 hours

(N1, N3, N8, A1, A2, A3, A4, A5, A6, A7, A17, A20, A21)

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use algebraic notation and symbols correctly;
- Know the difference between a term, expression, equation, formula and an identity;
- Write and manipulate an expression by collecting like terms;
- Substitute positive and negative numbers into expressions such as $3x + 4$ and $2x^3$ and then into expressions involving brackets and powers;
- Substitute numbers into formulae from mathematics and other subject using simple linear formulae, e.g. $l \times w$, $v = u + at$;
- Simplify expressions by cancelling, e.g. $\frac{4x}{2} = 2x$;
- Use instances of index laws for positive integer powers including when multiplying or dividing algebraic terms;
- Use instances of index laws, including use of zero, fractional and negative powers;
- Multiply a single term over a bracket and recognise factors of algebraic terms involving single brackets and simplify expressions by factorising, including subsequently collecting like terms;
- Expand the product of two linear expressions, i.e. double brackets working up to negatives in both brackets and also similar to $(2x + 3y)(3x - y)$;
- Know that squaring a linear expression is the same as expanding double brackets;
- Factorise quadratic expressions of the form $ax^2 + bx + c$;
- Factorise quadratic expressions using the difference of two squares;
- Set up simple equations from word problems and derive simple formulae;
- Understand the ≠ symbol (not equal), e.g. $6x + 4 \neq 3(x + 2)$, and introduce identity ≡ sign;
- Solve linear equations, with integer coefficients, in which the unknown appears on either side or on both sides of the equation;
- Solve linear equations which contain brackets, including those that have negative signs occurring anywhere in the equation, and those with a negative solution;
- Solve linear equations in one unknown, with integer or fractional coefficients;
- Set up and solve linear equations to solve to solve a problem;
- Derive a formula and set up simple equations from word problems, then solve these equations, interpreting the solution in the context of the problem;
- Substitute positive and negative numbers into a formula, solve the resulting equation including brackets, powers or standard form;
- Use and substitute formulae from mathematics and other subjects, including the kinematics formulae $v = u + at$, $v^2 - u^2 = 2as$, and $s = ut + \frac{1}{2}at^2$;
- Change the subject of a simple formula, i.e. linear one-step, such as $x = 4y$;
- Change the subject of a formula, including cases where the subject is on both sides of the original formula, or involving fractions and small powers of the subject;
- Simple proofs and use of ≡ in "show that" style questions; know the difference between an equation and an identity;
- Use iteration to find approximate solutions to equations, for simple equations in the first instance, then quadratic and cubic equations.

POSSIBLE SUCCESS CRITERIA

Simplify $4p - 2q^2 + 1 - 3p + 5q^2$.

Evaluate $4x^2 - 2x$ when $x = -5$.

Simplify $z^4 \times z^3$, $y^3 \div y^2$, $(a^7)^2$, $\left(8x^6y^4\right)^{\frac{1}{3}}$.

Expand and simplify $3(t - 1) + 57$.

Factorise $15x^2y - 35x^2y^2$.

Expand and simplify $(3x + 2)(4x - 1)$.

Factorise $6x^2 - 7x + 1$.

A room is 2 m longer than it is wide. If its area is 30 m^2 what is its perimeter?

Use fractions when working in algebraic situations.

Substitute positive and negative numbers into formulae.

Be aware of common scientific formulae.

Know the meaning of the 'subject' of a formula.

Change the subject of a formula when one step is required.

Change the subject of a formula when two steps are required.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Forming and solving equations involving algebra and other areas of mathematics such as area and perimeter.

Evaluate statements and justify which answer is correct by providing a counter-argument by way of a correct solution.

COMMON MISCONCEPTIONS

When expanding two linear expressions, poor number skills involving negatives and times tables will become evident.

Hierarchy of operations applied in the wrong order when changing the subject of a formula.

$a^0 = 0$.

$3xy$ and $5yx$ are different "types of term" and cannot be "collected" when simplifying expressions.

The square and cube operations on a calculator may not be similar on all makes.

Not using brackets with negative numbers on a calculator.

Not writing down all the digits on the display.

NOTES

Some of this will be a reminder from Key Stage 3 and could be introduced through investigative material such as handshake, frogs etc.

Students will have encountered much of this before and you may wish to introduce solving equations using function machines.

Practise factorisation where more than one variable is involved. NB More complex quadratics are covered in a later unit.

Plenty of practice should be given for factorising, and reinforce the message that making mistakes with negatives and times tables is a different skill to that being developed. Encourage students to expand linear sequences prior to simplifying when dealing with "double brackets".

Emphasise good use of notation.

Students need to realise that not all linear equations can be solved by observation or trial and improvement, and hence the use of a formal method is important.

Students can leave their answer in fraction form where appropriate. Emphasise that fractions are more accurate in calculations than rounded percentage or decimal equivalents.

Use examples involving formulae for circles, spheres, cones and kinematics when changing the subject of a formula.

For substitution use the distance–time–speed formula, and include speed of light given in standard form.

Students should be encouraged to use their calculator effectively by using the replay and ANS/EXE functions; reinforce the use of brackets and only rounding their final answer with trial and improvement.

2b. Sequences	**Teaching time**
(N8, N9, A23, A24, A25)	3-5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise simple sequences including at the most basic level odd, even, triangular, square and cube numbers and Fibonacci-type sequences (including those involving numbers in standard form or index form);
- Generate sequences of numbers, squared integers and sequences derived from diagrams;
- Describe in words a term-to-term sequence and identify which terms cannot be in a sequence;
- Generate specific terms in a sequence using the position-to-term rule and term-to-term rule;
- Find and use (to generate terms) the nth term of an arithmetic sequence;
- Use the nth term of an arithmetic sequence to decide if a given number is a term in the sequence, or find the first term above or below a given number;
- Identify which terms cannot be in a sequence by finding the nth term;
- Continue a quadratic sequence and use the nth term to generate terms;
- Find the nth term of quadratic sequences;
- Distinguish between arithmetic and geometric sequences;
- Use finite/infinite and ascending/descending to describe sequences;
- Recognise and use simple geometric progressions (rn where n is an integer, and r is a rational number > 0 or a surd);
- Continue geometric progression and find term to term rule, including negative, fraction and decimal terms;
- Solve problems involving sequences from real life situations.

POSSIBLE SUCCESS CRITERIA

Given a sequence, 'which is the 1st term greater than 50?'
Be able to solve problems involving sequences from real-life situations, such as:

- 1 grain of rice on first square, 2 grains on second, 4 grains on third, etc (geometric progression), or person saves £10 one week, £20 the next, £30 the next, etc;
- What is the amount of money after x months saving the same amount, or the height of tree that grows 6 m per year;
- Compare two pocket money options, e.g. same number of £ per week as your age from 5 until 21, or starting with £5 a week aged 5 and increasing by 15% a year until 21.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Evaluate statements about whether or not specific numbers or patterns are in a sequence and justify the reasons.

COMMON MISCONCEPTIONS

Students struggle to relate the position of the term to “n”.

NOTES

Emphasise use of $3n$ meaning 3 x n.
Students need to be clear on the description of the pattern in words, the difference between the terms and the algebraic description of the nth term.

UNIT 3: Averages and range, collecting data, representing data

Return to Overview

SPECIFICATION REFERENCES

G14 use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc.)

S2 interpret and construct tables, charts and diagrams, including frequency tables, bar charts, pie charts and pictograms for categorical data, vertical line charts for ungrouped discrete numerical data, tables and line graphs for time series data and know their appropriate use

S3 **construct and interpret diagrams for grouped discrete data and continuous data i.e. histograms with equal and unequal class intervals …**

S4 interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:

- appropriate graphical representation involving discrete, continuous and grouped data …
- appropriate measures of central tendency (median, mode and modal class) and spread (range, including consideration of outliers) …

S5 apply statistics to describe a population

S6 use and interpret scatter graphs of bivariate data; recognise correlation and know that it does not indicate causation; draw estimated lines of best fit; make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing

PRIOR KNOWLEDGE

Students should be able to read scales on graphs, draw circles, measure angles and plot coordinates in the first quadrant.
Students should have experience of tally charts.
Students will have used inequality notation.
Students must be able to find midpoint of two numbers.

KEYWORDS

Mean, median, mode, range, average, discrete, continuous, qualitative, quantitative, data, scatter graph, line of best fit, correlation, positive, negative, sample, population, stem and leaf, frequency, table, sort, pie chart, estimate

3a. Averages and range (G14, S2, S3, S4, S5)	**Teaching time** 3-5 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Design and use two-way tables for discrete and grouped data;
- Use information provided to complete a two-way table;
- Sort, classify and tabulate data and discrete or continuous quantitative data;
- Calculate mean and range, find median and mode from a small data set;
- Use a spreadsheet to calculate mean and range, and find median and mode;
- Recognise the advantages and disadvantages between measures of average;
- Construct and interpret stem and leaf diagrams (including back-to-back diagrams):
    - find the mode, median, range, as well as the greatest and least values from stem and leaf diagrams, and compare two distributions from stem and leaf diagrams (mode, median, range);
- Calculate the mean, mode, median and range from a frequency table (discrete data);
- Construct and interpret grouped frequency tables for continuous data:
    - for grouped data, find the interval which contains the median and the modal class;
    - estimate the mean with grouped data;
    - understand that the expression 'estimate' will be used where appropriate, when finding the mean of grouped data using mid-interval values.

POSSIBLE SUCCESS CRITERIA

Be able to state the median, mode, mean and range from a small data set.
Extract the averages from a stem and leaf diagram.
Estimate the mean from a table.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be able to provide reasons for choosing to use a specific average to support a point of view.
Given the mean, median and mode of five positive whole numbers, can you find the numbers?
Students should be able to provide a correct solution as a counter-argument to statements involving the "averages", e.g. Susan states that the median is 15, she is wrong. Explain why.

COMMON MISCONCEPTIONS

Students often forget the difference between continuous and discrete data.
Often the $\sum(m \times f)$ is divided by the number of classes rather than $\sum f$ when estimating the mean.

NOTES

Encourage students to cross out the midpoints of each group once they have used these numbers to in $m \times f$. This helps students to avoid summing m instead of f.
Remind students how to find the midpoint of two numbers.
Emphasise that continuous data is measured, i.e. length, weight, and discrete data can be counted, i.e. number of shoes.
Designing and using data collection is no longer in the specification, but may remain a useful topic as part of the overall data handling process.

3b. Representing and interpreting data and scatter graphs (S2, S3, S4, S5, S6)	**Teaching time** 4-6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Know which charts to use for different types of data sets;
- Produce and interpret composite bar charts;
- Produce and interpret comparative and dual bar charts;
- Produce and interpret pie charts:
  - find the mode and the frequency represented by each sector;
  - compare data from pie charts that represent different-sized samples;
- Produce and interpret frequency polygons for grouped data:
  - from frequency polygons, read off frequency values, compare distributions, calculate total population, mean, estimate greatest and least possible values (and range);
- Produce frequency diagrams for grouped discrete data:
  - read off frequency values, calculate total population, find greatest and least values;
- Produce histograms with equal class intervals:
  - estimate the median from a histogram with equal class width or any other information, such as the number of people in a given interval;
- Produce line graphs:
  - read off frequency values, calculate total population, find greatest and least values;
- Construct and interpret time–series graphs, comment on trends;
- Compare the mean and range of two distributions, or median or mode as appropriate;
- Recognise simple patterns, characteristics relationships in bar charts, line graphs and frequency polygons;
- Draw and interpret scatter graphs in terms of the relationship between two variables;
- Draw lines of best fit by eye, understanding what these represent;
- Identify outliers and ignore them on scatter graphs;
- Use a line of best fit, or otherwise, to predict values of a variable given values of the other variable;
- Distinguish between positive, negative and zero correlation using lines of best fit, and interpret correlation in terms of the problem;
- Understand that correlation does not imply causality, and appreciate that correlation is a measure of the strength of the association between two variables and that zero correlation does not necessarily imply 'no relationship' but merely 'no linear correlation';
- Explain an isolated point on a scatter graph;
- Use the line of best fit make predictions; interpolate and extrapolate apparent trends whilst knowing the dangers of so doing.

POSSIBLE SUCCESS CRITERIA

Use a time–series data graph to make a prediction about a future value.
Explain why same-size sectors on pie charts with different data sets do not represent the same number of items, but do represent the same proportion.
Make comparisons between two data sets.
Be able to justify an estimate they have made using a line of best fit.
Identify outliers and explain why they may occur.
Given two sets of data in a table, model the relationship and make predictions.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Many real-life situations that give rise to two variables provide opportunities for students to extrapolate and interpret the resulting relationship (if any) between the variables.
Choose which type of graph or chart to use for a specific data set and justify its use.
Evaluate statements in relation to data displayed in a graph/chart.

COMMON MISCONCEPTIONS

Students often forget the difference between continuous and discrete data.
Lines of best fit are often forgotten, but correct answers still obtained by sight.

NOTES

Interquartile range is covered in unit 14.
Misleading graphs are a useful activity for covering AO2 strand 5: Critically evaluate a given way of presenting information.
When doing time–series graphs, use examples from science, geography.
NB Moving averages are not explicitly mentioned in the programme of study but may be worth covering too.
Students need to be constantly reminded of the importance of drawing a line of best fit.
A possible extension includes drawing the line of best fit through the mean point (mean of x, mean of y).

UNIT 4: Fractions, percentages, ratio and proportion

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SPECIFICATION REFERENCES

N1 order positive and negative integers, decimals and fractions; …
N2 apply the four operations, including formal written methods, to integers, decimals and simple fractions (proper and improper), and mixed numbers – all both positive and negative; …
N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); use conventional notation for priority of operations, including brackets, powers, roots and reciprocals
N8 calculate exactly with fractions …
N10 work interchangeably with terminating decimals and their corresponding fractions (such as 3.5 and $\frac{7}{2}$ or 0.375 and $\frac{3}{8}$); **change recurring decimals into their corresponding fractions and vice versa**
N11 identify and work with fractions in ratio problems
N12 interpret fractions and percentages as operators
N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate
R2 use scale factors, scale diagrams and maps
R3 express one quantity as a fraction of another, where the fraction is less than 1 or greater than 1
R4 use ratio notation, including reduction to simplest form
R5 divide a given quantity into two parts in a given part:part or whole:part ratio; express the division of a quantity into two parts as a ratio; apply ratio to real contexts and problems (such as those involving conversion, comparison, scaling, mixing, concentrations)
R6 express a multiplicative relationship between two quantities as a ratio or a fraction
R7 understand and use proportion as equality of ratios
R8 relate ratios to fractions and to linear functions
R9 define percentage as 'number of parts per hundred'; interpret percentages and percentage changes as a fraction or a decimal, and interpret these multiplicatively; express one quantity as a percentage of another; compare two quantities using percentages; work with percentages greater than 100%; solve problems involving percentage change, including percentage increase/decrease, and original value problems and simple interest including in financial mathematics
R10 solve problems involving direct proportion; …

PRIOR KNOWLEDGE

Students should know the four operations of number.
Students should be able to find common factors.
Students should have a basic understanding of fractions as being 'parts of a whole'.
Students can define percentage as 'number of parts per hundred'.
Students are aware that percentages are used in everyday life.

KEYWORDS

Addition, subtraction, multiplication, division, fractions, mixed, improper, recurring, reciprocal, integer, decimal, termination, percentage, VAT, increase, decrease, multiplier, profit, loss, ratio, proportion, share, parts

4a. Fractions and percentages (N1, N2, N3, N8, N10 ,N12, N13, R3, R9)	**Teaching time** 11-13 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Express a given number as a fraction of another;
- Find equivalent fractions and compare the size of fractions;
- Write a fraction in its simplest form, including using it to simplify a calculation, e.g. $50 \div 20 = \frac{50}{20} = \frac{5}{2} = 2.5$;
- Find a fraction of a quantity or measurement, including within a context;
- Convert a fraction to a decimal to make a calculation easier;
- Convert between mixed numbers and improper fractions;
- Add and subtract fractions, including mixed numbers;
- Multiply and divide fractions, including mixed numbers and whole numbers and vice versa;
- Understand and use unit fractions as multiplicative inverses;
- By writing the denominator in terms of its prime factors, decide whether fractions can be converted to recurring or terminating decimals;
- Convert a fraction to a recurring decimal and vice versa;
- Find the reciprocal of an integer, decimal or fraction;
- Convert between fractions, decimals and percentages;
- Express a given number as a percentage of another number;
- Express one quantity as a percentage of another where the percentage is greater than 100%
- Find a percentage of a quantity;
- Find the new amount after a percentage increase or decrease;
- Work out a percentage increase or decrease, including: simple interest, income tax calculations, value of profit or loss, percentage profit or loss;
- Compare two quantities using percentages, including a range of calculations and contexts such as those involving time or money;
- Find a percentage of a quantity using a multiplier and use a multiplier to increase or decrease by a percentage in any scenario where percentages are used;
- Find the original amount given the final amount after a percentage increase or decrease (reverse percentages), including VAT;
- Use calculators for reverse percentage calculations by doing an appropriate division;
- Use percentages in real-life situations, including percentages greater than 100%;
- Describe percentage increase/decrease with fractions, e.g. 150% increase means $2\frac{1}{2}$ times as big;
- Understand that fractions are more accurate in calculations than rounded percentage or decimal equivalents, and choose fractions, decimals or percentages appropriately for calculations.

POSSIBLE SUCCESS CRITERIA

Express a given number as a fraction of another, including where the fraction is, for example, greater than 1, e.g. $\frac{120}{100} = 1\frac{2}{10} = 1\frac{1}{5}$.

Answer the following: James delivers 56 newspapers. $\frac{3}{8}$ of the newspapers have a magazine.

How many of the newspapers have a magazine?

Prove whether a fraction is terminating or recurring.

Convert a fraction to a decimal including where the fraction is greater than 1.
Be able to work out the price of a deposit, given the price of a sofa is £480 and the deposit is 15% of the price, without a calculator.
Find fractional percentages of amounts, with and without using a calculator.
Convince me that 0.125 is $\frac{1}{8}$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Many of these topics provide opportunities for reasoning in real-life contexts, particularly percentages:

- Calculate original values and evaluate statements in relation to this value justifying which statement is correct.

COMMON MISCONCEPTIONS

The larger the denominator, the larger the fraction.

Incorrect links between fractions and decimals, such as thinking that $\frac{1}{5}$ = 0.15, 5% = 0.5, 4% = 0.4, etc.
It is not possible to have a percentage greater than 100%.

NOTES

Ensure that you include fractions where only one of the denominators needs to be changed, in addition to where both need to be changed for addition and subtraction.
Include multiplying and dividing integers by fractions.
Use a calculator for changing fractions into decimals and look for patterns.
Recognise that every terminating decimal has its fraction with a 2 and/or 5 as a common factor in the denominator.
Use long division to illustrate recurring decimals.
Amounts of money should always be rounded to the nearest penny.
Encourage use of the fraction button.
Students should be reminded of basic percentages.
Amounts of money should always be rounded to the nearest penny, except where successive calculations are done (i.e. compound interest, which is covered in a later unit).
Emphasise the use of percentages in real-life situations.

4b. Ratio and proportion (N11, N12, N13, R2, R3, R4, R5, R6, R7, R8, R10)	**Teaching time** 5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Express the division of a quantity into a number parts as a ratio;
- Write ratios in form 1 : m or m : 1 and to describe a situation;
- Write ratios in their simplest form, including three-part ratios;
- Divide a given quantity into two or more parts in a given part : part or part : whole ratio;
- Use a ratio to find one quantity when the other is known;
- Write a ratio as a fraction and as a linear function;
- Identify direct proportion from a table of values, by comparing ratios of values;
- Use a ratio to compare a scale model to real-life object;
- Use a ratio to convert between measures and currencies, e.g. £1.00 = €1.36;
- Scale up recipes;
- Convert between currencies.

POSSIBLE SUCCESS CRITERIA

Write/interpret a ratio to describe a situation such as 1 blue for every 2 red …, 3 adults for every 10 children …
Recognise that two paints mixed red to yellow 5 : 4 and 20 : 16 are the same colour.
When a quantity is split in the ratio 3:5, what fraction does each person get?
Find amounts for three people when amount for one given.
Express the statement 'There are twice as many girls as boys' as the ratio 2 : 1 or the linear function $y = 2x$, where x is the number of boys and y is the number of girls.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems involving sharing in a ratio that include percentages rather than specific numbers such can provide links with other areas of Mathematics:

> In a youth club the ratio of the number of boys to the number of girls is 3 : 2 . 30% of the boys are under the age of 14 and 60% of the girls are under the age of 14. What percentage of the youth club is under the age of 14?

COMMON MISCONCEPTIONS

Students often identify a ratio-style problem and then divide by the number given in the question, without fully understanding the question.

NOTES

Three-part ratios are usually difficult for students to understand.
Also include using decimals to find quantities.
Use a variety of measures in ratio and proportion problems.
Include metric to imperial and vice versa, but give them the conversion factor, e.g. 5 miles = 8 km, 1 inch = 2.4 cm – these aren't specifically in the programme of study but are still useful.

UNIT 5: Angles, polygons, parallel lines; Right-angled triangles: Pythagoras and trigonometry

Return to Overview

SPECIFICATION REFERENCES

N7 Calculate with roots and with integer **and fractional** indices
N8 calculate exactly with fractions and **surds** …
N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); …
A4 simplify and manipulate algebraic expressions (including those involving surds) by collecting like terms …
A5 understand and use standard mathematical formulae; …
R12 compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors
G1 use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; …
G3 … understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)
G4 derive and apply the properties and definitions of: special types of quadrilaterals, including square, rectangle, parallelogram, trapezium, kite and rhombus; …
G6 apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including Pythagoras' theorem and the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs
G11 solve geometrical problems on coordinate axes
G20 know the formulae for: Pythagoras' theorem $a^2 + b^2 = c^2$, and the trigonometric ratios sine, cosine and tan; apply them to find angles and lengths in right-angled triangles … and in two dimensional figures
G21 know the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°

PRIOR KNOWLEDGE

Students should be able to rearrange simple formulae and equations, as preparation for rearranging trig formulae.
Students should recall basic angle facts.
Students should understand that fractions are more accurate in calculations than rounded percentage or decimal equivalents.

KEYWORDS

Quadrilateral, angle, polygon, interior, exterior, proof, tessellation, symmetry, parallel, corresponding, alternate, co-interior, vertices, edge, face, sides, Pythagoras' Theorem, sine, cosine, tan, trigonometry, opposite, hypotenuse, adjacent, ratio, elevation, depression, segment, length

5a. Polygons, angles and parallel lines (G1, G3, G4, G6, G11)	**Teaching time** 5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Classify quadrilaterals by their geometric properties and distinguish between scalene, isosceles and equilateral triangles;
- Understand 'regular' and 'irregular' as applied to polygons;
- Understand the proof that the angle sum of a triangle is 180°, and derive and use the sum of angles in a triangle;
- Use symmetry property of an isosceles triangle to show that base angles are equal;
- Find missing angles in a triangle using the angle sum in a triangle AND the properties of an isosceles triangle;
- Understand a proof of, and use the fact that, the exterior angle of a triangle is equal to the sum of the interior angles at the other two vertices;
- Explain why the angle sum of a quadrilateral is 360°; use the angle properties of quadrilaterals and the fact that the angle sum of a quadrilateral is 360°;
- Understand and use the angle properties of parallel lines and find missing angles using the properties of corresponding and alternate angles, giving reasons;
- Use the angle sums of irregular polygons;
- Calculate and use the sums of the interior angles of polygons; use the sum of angles in a triangle and use the angle sum in any polygon to derive the properties of regular polygons;
- Use the sum of the exterior angles of any polygon is 360°;
- Use the sum of the interior angles of an n-sided polygon;
- Use the sum of the interior angle and the exterior angle is 180°;
- Find the size of each interior angle, or the size of each exterior angle, or the number of sides of a regular polygon, and use the sum of angles of irregular polygons;
- Calculate the angles of regular polygons and use these to solve problems;
- Use the side/angle properties of compound shapes made up of triangles, lines and quadrilaterals, including solving angle and symmetry problems for shapes in the first quadrant, more complex problems and using algebra;
- Use angle facts to demonstrate how shapes would 'fit together', and work out interior angles of shapes in a pattern.

POSSIBLE SUCCESS CRITERIA

Name all quadrilaterals that have a specific property.
Given the size of its exterior angle, how many sides does the polygon have?
What is the same, and what is different between families of polygons?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step "angle chasing"-style problems that involve justifying how students have found a specific angle will provide opportunities to develop a chain of reasoning.
Geometrical problems involving algebra whereby equations can be formed and solved allow students the opportunity to make and use connections with different parts of mathematics.

COMMON MISCONCEPTIONS

Some students will think that all trapezia are isosceles, or a square is only square if 'horizontal', or a 'non-horizontal' square is called a diamond.
Pupils may believe, incorrectly, that:

- perpendicular lines have to be horizontal/vertical;
- all triangles have rotational symmetry of order 3;
- all polygons are regular.

Incorrectly identifying the 'base angles' (i.e. the equal angles) of an isosceles triangle when not drawn horizontally.

NOTES

Demonstrate that two line segments that do not meet could be perpendicular – if they are extended and they would meet at right angles.
Students must be encouraged to use geometrical language appropriately, 'quote' the appropriate reasons for angle calculations and show step-by-step deduction when solving multi-step problems.
Emphasise that diagrams in examinations are seldom drawn accurately.
Use tracing paper to show which angles in parallel lines are equal.
Students must use co-interior, not supplementary, to describe paired angles inside parallel lines. (NB Supplementary angles are any angles that add to 180, not specifically those in parallel lines.)
Use triangles to find angle sums of polygons; this could be explored algebraically as an investigation.

5b. Pythagoras’ Theorem and trigonometry (N7, N8, N15, A4, A5, R12, G6, G20, G21)	**Teaching time** 5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand, recall and use Pythagoras’ Theorem in 2D;
- Given three sides of a triangle, justify if it is right-angled or not;
- Calculate the length of the hypotenuse in a right-angled triangle (including decimal lengths and a range of units);
- Find the length of a shorter side in a right-angled triangle;
- Calculate the length of a line segment AB given pairs of points;
- Give an answer to the use of Pythagoras’ Theorem in surd form;
- Understand, use and recall the trigonometric ratios sine, cosine and tan, and apply them to find angles and lengths in general triangles in 2D figures;
- Use the trigonometric ratios to solve 2D problems;
- Find angles of elevation and depression;
- Know the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°.

POSSIBLE SUCCESS CRITERIA

Does 2, 3, 6 give a right-angled triangle?
Justify when to use Pythagoras’ Theorem and when to use trigonometry.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Combined triangle problems that involve consecutive application of Pythagoras’ Theorem or a combination of Pythagoras’ Theorem and the trigonometric ratios.
In addition to abstract problems, students should be encouraged to apply Pythagoras’ Theorem and/or the trigonometric ratios to real-life scenarios that require them to evaluate whether their answer fulfils certain criteria, e.g. the angle of elevation of 6.5 m ladder cannot exceed 65°. What is the greatest height it can reach? Rounding skills will be important here when justifying their findings.

COMMON MISCONCEPTIONS

Answers may be displayed on a calculator in surd form.
Students forget to square root their final answer, or round their answer prematurely.

NOTES

Students may need reminding about surds.
Drawing the squares on the three sides will help when deriving the rule.
Scale drawings are not acceptable.
Calculators need to be in degree mode.
To find in right-angled triangles the exact values of sin θ and cos θ for θ = 0°, 30°, 45°, 60° and 90°, use triangles with angles of 30°, 45° and 60°.
Use a suitable mnemonic to remember SOHCAHTOA.
Use Pythagoras’ Theorem and trigonometry together.

UNIT 6: Real-life and algebraic linear graphs, quadratic and cubic graphs, the equation of a circle, plus rates of change and area under graphs made from straight lines

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SPECIFICATION REFERENCES

N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate

A8 work with coordinates in all four quadrants

A9 plot graphs of equations that correspond to straight-line graphs in the coordinate plane; use the form $y = mx + c$ to identify parallel **and perpendicular lines**; find the equation of the line through two given points, or through one point with a given gradient

A10 identify and interpret gradients and intercepts of linear functions graphically and algebraically

A11 identify and interpret roots, intercepts, turning points of quadratic functions graphically; …

A12 recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, …

A14 plot and interpret … graphs of non-standard functions in real contexts to find approximate solutions to problems such as simple kinematic problems involving distance, speed and acceleration

A15 **calculate or estimate gradients of graphs and areas under graphs (including quadratic and non-linear graphs) and interpret results in cases such as distance–time graphs, velocity–time graphs … (this does not include calculus)**

A16 **recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point**

A17 solve linear equations in one unknown … (including those with the unknown on both sides of the equation); find approximate solutions using a graph

A18 solve quadratic equations (**including those that require rearrangement**) algebraically by factorising, **by completing the square and by using the quadratic formula**; find approximate solutions using a graph

R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts

R10 solve problems involving direct … proportion, including graphical … representations

R11 use compound units such as speed, … unit pricing, …

R14 … recognise and interpret graphs that illustrate direct and inverse proportion

PRIOR KNOWLEDGE

Students can identify coordinates of given points in the first quadrant or all four quadrants.
Students can use Pythagoras' Theorem and calculate the area of compound shapes.
Students can use and draw conversion graphs for these units.
Students can use function machines and inverse operations.

KEYWORDS

Coordinate, axes, 3D, Pythagoras, graph, speed, distance, time, velocity, quadratic, solution, root, function, linear, circle, cubic, approximate, gradient, perpendicular, parallel, equation

6a. Graphs: the basics and real-life graphs (N13, A8, A9, A10, A14, A15, R1, R11)

Teaching time
5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Identify and plot points in all four quadrants;
- Draw and interpret straight-line graphs for real-life situations, including ready reckoner graphs, conversion graphs, fuel bills, fixed charge and cost per item;
- Draw distance–time and velocity–time graphs;
- Use graphs to calculate various measures (of individual sections), including: unit price (gradient), average speed, distance, time, acceleration; including using enclosed areas by counting squares or using areas of trapezia, rectangles and triangles;
- Find the coordinates of the midpoint of a line segment with a diagram given and coordinates;
- Find the coordinates of the midpoint of a line segment from coordinates;
- Calculate the length of a line segment given the coordinates of the end points;
- Find the coordinates of points identified by geometrical information.
- Find the equation of the line through two given points.

POSSIBLE SUCCESS CRITERIA

Interpret a description of a journey into a distance–time or speed–time graph.
Calculate various measures given a graph.
Calculate an end point of a line segment given one coordinate and its midpoint.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Speed/distance graphs can provide opportunities for interpreting non-mathematical problems as a sequence of mathematical processes, whilst also requiring students to justify their reasons why one vehicle is faster than another.
Calculating the length of a line segment provides links with other areas of mathematics.

COMMON MISCONCEPTIONS

Where line segments cross the y-axis, finding midpoints and lengths of segments is particularly challenging as students have to deal with negative numbers.

NOTES

Careful annotation should be encouraged: it is good practice to label the axes and check that students understand the scales.
Use various measures in the distance–time and velocity–time graphs, including miles, kilometres, seconds, and hours, and include large numbers in standard form.
Ensure that you include axes with negative values to represent, for example, time before present time, temperature or depth below sea level.
Metric-to-imperial measures are not specifically included in the programme of study, but it is a useful skill and ideal for conversion graphs.
Emphasise that velocity has a direction.
Coordinates in 3D can be used to extend students.

6b. Linear graphs and coordinate geometry (A9, A10, A12, A17, R10, R14)	Teaching time 7-9 hours

OBJECTIVES

By the end of the unit, students should be able to:

- Plot and draw graphs of $y = a$, $x = a$, $y = x$ and $y = -x$, drawing and recognising lines parallel to axes, plus $y = x$ and $y = -x$;
- Identify and interpret the gradient of a line segment;
- Recognise that equations of the form $y = mx + c$ correspond to straight-line graphs in the coordinate plane;
- Identify and interpret the gradient and y-intercept of a linear graph given by equations of the form $y = mx + c$;
- Find the equation of a straight line from a graph in the form $y = mx + c$;
- Plot and draw graphs of straight lines of the form $y = mx + c$ with and without a table of values;
- Sketch a graph of a linear function, using the gradient and y-intercept (i.e. without a table of values);
- Find the equation of the line through one point with a given gradient;
- Identify and interpret gradient from an equation $ax + by = c$;
- Find the equation of a straight line from a graph in the form $ax + by = c$;
- Plot and draw graphs of straight lines in the form $ax + by = c$;
- Interpret and analyse information presented in a range of linear graphs:
  - use gradients to interpret how one variable changes in relation to another;
  - find approximate solutions to a linear equation from a graph;
  - identify direct proportion from a graph;
  - find the equation of a line of best fit (scatter graphs) to model the relationship between quantities;
- Explore the gradients of parallel lines and lines perpendicular to each other;
- Interpret and analyse a straight-line graph and generate equations of lines parallel and perpendicular to the given line;
- Select and use the fact that when $y = mx + c$ is the equation of a straight line, then the gradient of a line parallel to it will have a gradient of m and a line perpendicular to this line will have a gradient of $-\frac{1}{m}$.

POSSIBLE SUCCESS CRITERIA

Find the equation of the line passing through two coordinates by calculating the gradient first.
Understand that the form $y = mx + c$ or $ax + by = c$ represents a straight line.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Given an equation of a line provide a counter argument as to whether or not another equation of a line is parallel or perpendicular to the first line.
Decide if lines are parallel or perpendicular without drawing them and provide reasons.

COMMON MISCONCEPTIONS

Students can find visualisation of a question difficult, especially when dealing with gradients resulting from negative coordinates.

NOTES

Encourage students to sketch what information they are given in a question – emphasise that it is a sketch.
Careful annotation should be encouraged – it is good practice to label the axes and check that students understand the scales.

6c. Quadratic, cubic and other graphs	**Teaching time**
(A11, A12, A14, A16, A18)	5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise a linear, quadratic, cubic, reciprocal and circle graph from its shape;
- Generate points and plot graphs of simple quadratic functions, then more general quadratic functions;
- Find approximate solutions of a quadratic equation from the graph of the corresponding quadratic function;
- Interpret graphs of quadratic functions from real-life problems;
- Draw graphs of simple cubic functions using tables of values;
- Interpret graphs of simple cubic functions, including finding solutions to cubic equations;
- Draw graphs of the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$ using tables of values;
- Draw circles, centre the origin, equation $x^2 + y^2 = r^2$.

POSSIBLE SUCCESS CRITERIA

Select and use the correct mathematical techniques to draw linear, quadratic, cubic and reciprocal graphs.
Identify a variety of functions by the shape of the graph.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Match equations of quadratics and cubics with their graphs by recognising the shape or by sketching.

COMMON MISCONCEPTIONS

Students struggle with the concept of solutions and what they represent in concrete terms.

NOTES

Use lots of practical examples to help model the quadratic function, e.g. draw a graph to model the trajectory of a projectile and predict when/where it will land.
Ensure axes are labelled and pencils used for drawing.
Graphical calculations or appropriate ICT will allow students to see the impact of changing variables within a function.

UNIT 7: Perimeter, area and volume, plane shapes and prisms, circles, cylinders, spheres, cones; Accuracy and bounds

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SPECIFICATION REFERENCES

N8 calculate exactly with … multiples of π; …
N14 estimate answers; check calculations using approximation and estimation, including answers obtained using technology
N15 round numbers and measures to an appropriate degree of accuracy (e.g. to a specified number of decimal places or significant figures); use inequality notation to specify simple error intervals due to truncation or rounding
N16 apply and interpret limits of accuracy, **including upper and lower bounds**
A5 understand and use standard mathematical formulae; rearrange formulae to change the subject
A21 translate simple situations or procedures into algebraic expressions or formulae; derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution
R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) … in numerical and algebraic contexts
G1 use conventional terms and notations: points, lines, vertices, edges, planes, parallel lines, perpendicular lines, right angles, polygons, regular polygons and polygons with reflection and/or rotation symmetries; …
G9 identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment
G12 identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres
G13 construct and interpret plans and elevations of 3D shapes.
G14 use standard units of measure and related concepts (length, area, volume/capacity, mass, time, money, etc)
G16 know and apply formulae to calculate: area of triangles, parallelograms, trapezia; volume of cuboids and other right prisms (including cylinders)
G17 know the formulae: circumference of a circle = $2\pi r = \pi d$, area of a circle = πr^2; calculate: perimeters of 2D shapes, including circles; areas of circles and composite shapes; surface area and volume of spheres, pyramids, cones and composite solids
G18 calculate arc lengths, angles and areas of sectors of circles

PRIOR KNOWLEDGE

Students should know the names and properties of 3D forms.
The concept of perimeter and area by measuring lengths of sides will be familiar to students.
Students should be able to substitute numbers into an equation and give answers to an appropriate degree of accuracy.
Students should know the various metric units.

KEYWORDS

Triangle, rectangle, parallelogram, trapezium, area, perimeter, formula, length, width, prism, compound, measurement, polygon, cuboid, volume, nets, isometric, symmetry, vertices, edge, face, circle, segment, arc, sector, cylinder, circumference, radius, diameter, pi, composite, sphere, cone, capacity, hemisphere, segment, frustum, bounds, accuracy, surface area

7a. Perimeter, area and circles	**Teaching time**
(N8, N14, N15, A5, R1, G1, G9, G14, G16, G17, G18)	4-6 hours

OBJECTIVES

By the end of the unit, students should be able to:

- Recall and use the formulae for the area of a triangle, rectangle, trapezium and parallelogram using a variety of metric measures;
- Calculate the area of compound shapes made from triangles, rectangles, trapezia and parallelograms using a variety of metric measures;
- Find the perimeter of a rectangle, trapezium and parallelogram using a variety of metric measures;
- Calculate the perimeter of compound shapes made from triangles and rectangles;
- Estimate area and perimeter by rounding measurements to 1 significant figure to check reasonableness of answers;
- Recall the definition of a circle and name and draw parts of a circle;
- Recall and use formulae for the circumference of a circle and the area enclosed by a circle (using circumference = $2\pi r = \pi d$ and area of a circle = πr^2) using a variety of metric measures;
- Use $\pi \approx 3.142$ or use the π button on a calculator;
- Calculate perimeters and areas of composite shapes made from circles and parts of circles (including semicircles, quarter-circles, combinations of these and also incorporating other polygons);
- Calculate arc lengths, angles and areas of sectors of circles;
- Find radius or diameter, given area or circumference of circles in a variety of metric measures;
- Give answers to an appropriate degree of accuracy or in terms of π;
- Form equations involving more complex shapes and solve these equations.

POSSIBLE SUCCESS CRITERIA

Calculate the area and/or perimeter of shapes with different units of measurement.
Understand that answers in terms of π are more accurate.
Calculate the perimeters and/or areas of circles, semicircles and quarter-circles given the radius or diameter and vice versa.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Using compound shapes or combinations of polygons that require students to subsequently interpret their result in a real-life context.
Know the impact of estimating their answers and whether it is an overestimate or underestimate in relation to a given context.
Multi-step problems, including the requirement to form and solve equations, provide links with other areas of mathematics.

COMMON MISCONCEPTIONS

Students often get the concepts of area and perimeter confused.
Shapes involving missing lengths of sides often result in incorrect answers.
Diameter and radius are often confused, and recollection of area and circumference of circles involves incorrect radius or diameter.

NOTES

Encourage students to draw a sketch where one isn't provided.
Emphasise the functional elements with carpets, tiles for walls, boxes in a larger box, etc. Best value and minimum cost can be incorporated too.
Ensure that examples use different metric units of length, including decimals.
Emphasise the need to learn the circle formulae; "Cherry Pie's Delicious" and "Apple Pies are too" are good ways to remember them.
Ensure that students know it is more accurate to leave answers in terms of π, but only when asked to do so.

7b. 3D forms and volume, cylinders, cones and spheres (N8, N14, N15, A5, A21, G1, G12, G13, G14, G16, G17)	**Teaching time** 6-8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Find the surface area of prisms using the formulae for triangles and rectangles, and other (simple) shapes with and without a diagram;
- Draw sketches of 3D solids and identify planes of symmetry of 3D solids, and sketch planes of symmetry;
- Recall and use the formula for the volume of a cuboid or prism made from composite 3D solids using a variety of metric measures;
- Convert between metric measures of volume and capacity, e.g. 1 ml = 1 cm^3;
- Use volume to solve problems;
- Estimating surface area, perimeter and volume by rounding measurements to 1 significant figure to check reasonableness of answers;
- Use $\pi \approx 3.142$ or use the π button on a calculator;
- Find the volume and surface area of a cylinder;
- Recall and use the formula for volume of pyramid;
- Find the surface area of a pyramid;
- Use the formulae for volume and surface area of spheres and cones;
- Solve problems involving more complex shapes and solids, including segments of circles and frustums of cones;
- Find the surface area and volumes of compound solids constructed from cubes, cuboids, cones, pyramids, spheres, hemispheres, cylinders;
- Give answers to an appropriate degree of accuracy or in terms of π;
- Form equations involving more complex shapes and solve these equations.

POSSIBLE SUCCESS CRITERIA

Given dimensions of a rectangle and a pictorial representation of it when folded, work out the dimensions of the new shape.
Work out the length given the area of the cross-section and volume of a cuboid.
Understand that answers in terms of π are more accurate.
Given two solids with the same volume and the dimensions of one, write and solve an equation in terms of π to find the dimensions of the other, e.g. a sphere is melted down to make ball bearings of a given radius, how many will it make?

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Combinations of 3D forms such as a cone and a sphere where the radius has to be calculated given the total height.

COMMON MISCONCEPTIONS

Students often get the concepts of surface area and volume confused.

NOTES

Encourage students to draw a sketch where one isn't provided.
Use lots of practical examples to ensure that students can distinguish between surface area and volume. Making solids using multi-link cubes can be useful.
Solve problems including examples of solids in everyday use.
Drawing 3D shapes in 2D using isometric grids isn't an explicit objective but provides an ideal introduction to the topic and for some students provides the scaffolding needed when drawing 3D solids.
Scaffold drawing 3D shapes by initially using isometric paper.
Whilst not an explicit objective, it is useful for students to draw and construct nets and show how they fold to make 3D solids, allowing students to make the link between 3D shapes and their nets. This will enable students to understand that there is often more than one net that can form a 3D shape.
Formulae for curved surface area and volume of a sphere, and surface area and volume of a cone will be given on the formulae page of the examinations.
Ensure that students know it is more accurate to leave answers in terms of π but only when asked to do so.

7c. Accuracy and bounds	**Teaching time**
(N15, N16)	4-6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Calculate the upper and lowers bounds of numbers given to varying degrees of accuracy;
- Calculate the upper and lower bounds of an expression involving the four operations;
- Find the upper and lower bounds in real-life situations using measurements given to appropriate degrees of accuracy;
- Find the upper and lower bounds of calculations involving perimeters, areas and volumes of 2D and 3D shapes;
- Calculate the upper and lower bounds of calculations, particularly when working with measurements;
- Use inequality notation to specify an error interval due to truncation or rounding.

POSSIBLE SUCCESS CRITERIA

Round 16,000 people to the nearest 1000.
Round 1100 g to 1 significant figure.
Work out the upper and lower bounds of a formula where all terms are given to 1 decimal place.
Be able to justify that measurements to the nearest whole unit may be inaccurate by up to one half in either direction.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

This sub-unit provides many opportunities for students to evaluate their answers and provide counter-arguments in mathematical and real-life contexts, in addition to requiring them to understand the implications of rounding their answers.

COMMON MISCONCEPTIONS

Students readily accept the rounding for lower bounds, but take some convincing in relation to upper bounds.

NOTES

Students should use 'half a unit above' and 'half a unit below' to find upper and lower bounds.
Encourage use a number line when introducing the concept.

UNIT 8: Transformations; Constructions: triangles, nets, plan and elevation, loci, scale drawings and bearings

Return to Overview

SPECIFICATION REFERENCES

R2 use scale factors, scale diagrams and maps

R6 express a multiplicative relationship between two quantities as a ratio or a fraction

G2 use the standard ruler and compass constructions (perpendicular bisector of a line segment, constructing a perpendicular to a given line from/at a given point, bisecting a given angle); use these to construct given figures and solve loci problems; know that the perpendicular distance from a point to a line is the shortest distance to the line

G3 apply the properties of angles at a point, angles at a point on a straight line, vertically opposite angles; understand and use alternate and corresponding angles on parallel lines; derive and use the sum of angles in a triangle (e.g. to deduce and use the angle sum in any polygon, and to derive properties of regular polygons)

G5 use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)

G7 identify, describe and construct congruent and similar shapes, including on a coordinate axis, by considering rotation, reflection, translation and enlargement (including fractional **and negative** scale factors)

G8 **describe the changes and invariance achieved by combinations of rotations, reflections and translations**

G12 identify properties of the faces, surfaces, edges and vertices of: cubes, cuboids, prisms, cylinders, pyramids, cones and spheres

G13 construct and interpret plans and elevations of 3D shapes

G15 measure line segments and angles in geometric figures, including interpreting maps and scale drawings and use of bearings

G24 describe translations as 2D vectors

G25 apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; …

PRIOR KNOWLEDGE

Students should be able to recognise 2D shapes.
Students should be able to plot coordinates in four quadrants and linear equations parallel to the coordinate axes.

KEYWORDS

Rotation, reflection, translation, transformation, enlargement, scale factor, vector, centre, angle, direction, mirror line, centre of enlargement, describe, distance, congruence, similar, combinations, single, corresponding, constructions, compasses, protractor, bisector, bisect, line segment, perpendicular, loci, bearing

8a. Transformations (R6, G5, G7, G8, G24, G25)	**Teaching time** 5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Distinguish properties that are preserved under particular transformations;
- Recognise and describe rotations – know that that they are specified by a centre and an angle;
- Rotate 2D shapes using the origin or any other point (not necessarily on a coordinate grid);
- Identify the equation of a line of symmetry;
- Recognise and describe reflections on a coordinate grid – know to include the mirror line as a simple algebraic equation, $x = a$, $y = a$, $y = x$, $y = -x$ and lines not parallel to the axes;
- Reflect 2D shapes using specified mirror lines including lines parallel to the axes and also $y = x$ and $y = -x$;
- Recognise and describe single translations using column vectors on a coordinate grid;
- Translate a given shape by a vector;
- Understand the effect of one translation followed by another, in terms of column vectors (to introduce vectors in a concrete way);
- Enlarge a shape on a grid without a centre specified;
- Describe and transform 2D shapes using enlargements by a positive integer, positive fractional, and negative scale factor;
- Know that an enlargement on a grid is specified by a centre and a scale factor;
- Identify the scale factor of an enlargement of a shape;
- Enlarge a given shape using a given centre as the centre of enlargement by counting distances from centre, and find the centre of enlargement by drawing;
- Find areas after enlargement and compare with before enlargement, to deduce multiplicative relationship (area scale factor); given the areas of two shapes, one an enlargement of the other, find the scale factor of the enlargement (whole number values only);
- Use congruence to show that translations, rotations and reflections preserve length and angle, so that any figure is congruent to its image under any of these transformations;
- Describe and transform 2D shapes using combined rotations, reflections, translations, or enlargements;
- Describe the changes and invariance achieved by combinations of rotations, reflections and translations.

POSSIBLE SUCCESS CRITERIA

Recognise similar shapes because they have equal corresponding angles and/or sides scaled up in same ratio.
Understand that translations are specified by a distance and direction (using a vector).
Recognise that enlargements preserve angle but not length.
Understand that distances and angles are preserved under rotations, reflections and translations so that any shape is congruent to its image.
Understand that similar shapes are enlargements of each other and angles are preserved.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be given the opportunity to explore the effect of reflecting in two parallel mirror lines and combining transformations.

COMMON MISCONCEPTIONS

Students often use the term ‘transformation’ when describing transformations instead of the required information.
Lines parallel to the coordinate axes often get confused.

NOTES

Emphasise the need to describe the transformations fully, and if asked to describe a ‘single’ transformation students should not include two types.
Find the centre of rotation, by trial and error and by using tracing paper. Include centres on or inside shapes.
Area of similar shapes is covered in unit 12.

8b. Constructions, loci and bearings (R2, G2, G3, G12, G13, G15)	Teaching time 6-8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Understand and draw front and side elevations and plans of shapes made from simple solids;
- Given the front and side elevations and the plan of a solid, draw a sketch of the 3D solid;
- Use and interpret maps and scale drawings, using a variety of scales and units;
- Read and construct scale drawings, drawing lines and shapes to scale;
- Estimate lengths using a scale diagram;
- Understand, draw and measure bearings;
- Calculate bearings and solve bearings problems, including on scaled maps, and find/mark and measure bearings
- Use the standard ruler and compass constructions:
  - bisect a given angle;
  - construct a perpendicular to a given line from/at a given point;
  - construct angles of 90°, 45°;
  - perpendicular bisector of a line segment;
- Construct:
  - a region bounded by a circle and an intersecting line;
  - a given distance from a point and a given distance from a line;
  - equal distances from two points or two line segments;
  - regions which may be defined by 'nearer to' or 'greater than';
- Find and describe regions satisfying a combination of loci, including in 3D;
- Use constructions to solve loci problems including with bearings;
- Know that the perpendicular distance from a point to a line is the shortest distance to the line.

POSSIBLE SUCCESS CRITERIA

Able to read and construct scale drawings.
When given the bearing of a point A from point B, can work out the bearing of B from A.
Know that scale diagrams, including bearings and maps, are 'similar' to the real-life examples.
Able to sketch the locus of point on a vertex of a rotating shape as it moves along a line, of a point on the circumference and at the centre of a wheel.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Interpret a given plan and side view of a 3D form to be able to produce a sketch of the form.
Problems involving combinations of bearings and loci can provide a rich opportunity to link with other areas of mathematics and allow students to justify their findings.

COMMON MISCONCEPTIONS

Correct use of a protractor may be an issue.

NOTES

Drawings should be done in pencil.
Relate loci problems to real-life scenarios, including mobile phone masts and coverage.
Construction lines should not be erased.

UNIT 9: Algebra: Solving quadratic equations and inequalities, solving simultaneous equations algebraically

Return to Overview

SPECIFICATION REFERENCES

N1 order positive and negative integers, decimals and fractions; use the symbols =, ≠, <, >, ≤, ≥

N8 calculate exactly with … **surds**; … **simplify surd expressions involving squares (e.g. √12 = √(4 × 3) = √4 × √3 = 2√3)**

A4 simplify and manipulate algebraic expressions (including those involving surds …) by: … **factorising quadratic expressions of the form $ax^2 + bx + c$**

A5 understand and use standard mathematical formulae; rearrange formulae to change the subject

A9 … find the equation of the line through two given points, or through one point with a given gradient

A11 identify and interpret roots … of quadratic functions algebraically …

A18 solve quadratic equations **(including those that require rearrangement)** algebraically by factorising, **by completing the square and by using the quadratic formula**; …

A19 solve two simultaneous equations in two variables (linear/linear **or linear/quadratic**) algebraically; find approximate solutions using a graph

A21 … derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution.

A22 solve linear inequalities in one **or two** variable**(s)**, **and quadratic inequalities in one variable**; represent the solution set on a number line, **using set notation and on a graph**

PRIOR KNOWLEDGE

Students should understand the ≥ and ≤ symbols.
Students can substitute into, solve and rearrange linear equations.
Students should be able to factorise simple quadratic expressions.
Students should be able to recognise the equation of a circle.

KEYWORDS

Quadratic, solution, root, linear, solve, simultaneous, inequality, completing the square, factorise, rearrange, surd, function, solve, circle, sets, union, intersection

9a. Solving quadratics and simultaneous equations	**Teaching time**
(N8, A4, A5, A9, A11, A18, A19, A21)	6-8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Factorise quadratic expressions in the form $ax^2 + bx + c$;
- Set up and solve quadratic equations;
- Solve quadratic equations by factorisation and completing the square;
- Solve quadratic equations that need rearranging;
- Solve quadratic equations by using the quadratic formula;
- Find the exact solutions of two simultaneous equations in two unknowns;
- Use elimination or substitution to solve simultaneous equations;
- Solve exactly, by elimination of an unknown, two simultaneous equations in two unknowns:
  - linear / linear, including where both need multiplying;
  - linear / quadratic;
  - linear / $x^2 + y^2 = r^2$;
- Set up and solve a pair of linear simultaneous equations in two variables, including to represent a situation;
- Interpret the solution in the context of the problem;

POSSIBLE SUCCESS CRITERIA

Solve $3x^2 + 4 = 100$.
Know that the quadratic formula can be used to solve all quadratic equations, and often provides a more efficient method than factorising or completing the square.
Have an understanding of solutions that can be written in surd form.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require students to set up and solve a pair of simultaneous equations in a real-life context, such as 2 adult tickets and 1 child ticket cost £28, and 1 adult ticket and 3 child tickets cost £34. How much does 1 adult ticket cost?

COMMON MISCONCEPTIONS

Using the formula involving negatives can result in incorrect answers.
If students are using calculators for the quadratic formula, they can come to rely on them and miss the fact that some solutions can be left in surd form.

NOTES

Remind students to use brackets for negative numbers when using a calculator, and remind them of the importance of knowing when to leave answers in surd form.
Link to unit 2, where quadratics were solved algebraically (when $a = 1$).
The quadratic formula must now be known; it will not be given in the exam paper.
Reinforce the fact that some problems may produce one inappropriate solution which can be ignored.
Clear presentation of working out is essential.
Link with graphical representations.

9b. Inequalities	**Teaching time**
(N1, A22)	5–7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Show inequalities on number lines;
- Write down whole number values that satisfy an inequality;
- Solve simple linear inequalities in one variable, and represent the solution set on a number line;
- Solve two linear inequalities in x, find the solution sets and compare them to see which value of x satisfies both solve linear inequalities in two variables algebraically;
- Use the correct notation to show inclusive and exclusive inequalities.

POSSIBLE SUCCESS CRITERIA

Use inequality symbols to compare numbers.
Given a list of numbers, represent them on a number line using the correct notation.
Solve equations involving inequalities.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that require student to justify why certain values in a solution can be ignored.

COMMON MISCONCEPTIONS

When solving inequalities students often state their final answer as a number quantity, and exclude the inequality or change it to =.
Some students believe that –6 is greater than –3.

NOTES

Emphasise the importance of leaving their answer as an inequality (and not changing it to =).
Link to units 2 and 9a, where quadratics and simultaneous equations were solved.
Students can leave their answers in fractional form where appropriate.
Ensure that correct language is used to avoid reinforcing misconceptions: for example, 0.15 should never be read as 'zero point fifteen', and 5 > 3 should be read as 'five is greater than 3', not '5 is bigger than 3'.

UNIT 10: Probability

Teaching Time
7-9 hours

Return to Overview

SPECIFICATION REFERENCES

N5 apply systematic listing strategies, **including use of the product rule for counting** …

P1 record, describe and analyse the frequency of outcomes of probability experiments using tables and frequency trees

P2 apply ideas of randomness, fairness and equally likely events to calculate expected outcomes of multiple future experiments

P3 relate relative expected frequencies to theoretical probability, using appropriate language and the 0–1 probability scale

P4 apply the property that the probabilities of an exhaustive set of outcomes sum to one; apply the property that the probabilities of an exhaustive set of mutually exclusive events sum to one

P5 understand that empirical unbiased samples tend towards theoretical probability distributions, with increasing sample size

P6 enumerate sets and combinations of sets systematically, using tables, grids, Venn diagrams and tree diagrams

P7 construct theoretical possibility spaces for single and combined experiments with equally likely outcomes and use these to calculate theoretical probabilities

P8 calculate the probability of independent and dependent combined events, including using tree diagrams and other representations, and know the underlying assumptions

P9 **calculate and interpret conditional probabilities through representation using expected frequencies with two-way tables, tree diagrams and Venn diagrams**

PRIOR KNOWLEDGE

Students should understand that a probability is a number between 0 and 1, and distinguish between events which are impossible, unlikely, even chance, likely, and certain to occur.
Students should be able to mark events and/or probabilities on a probability scale of 0 to 1.
Students should know how to add and multiply fractions and decimals.
Students should have experience of expressing one number as a fraction of another number.

KEYWORDS

Probability, mutually exclusive, conditional, tree diagrams, sample space, outcomes, theoretical, relative frequency, Venn diagram, fairness, experimental

OBJECTIVES

By the end of the unit, students should be able to:

- Write probabilities using fractions, percentages or decimals;
- Understand and use experimental and theoretical measures of probability, including relative frequency to include outcomes using dice, spinners, coins, etc;
- Estimate the number of times an event will occur, given the probability and the number of trials;
- Find the probability of successive events, such as several throws of a single dice;
- List all outcomes for single events, and combined events, systematically;
- Draw sample space diagrams and use them for adding simple probabilities;
- Know that the sum of the probabilities of all outcomes is 1;

- Use 1 – p as the probability of an event not occurring where p is the probability of the event occurring;
- Work out probabilities from Venn diagrams to represent real-life situations and also ‘abstract’ sets of numbers/values;
- Use union and intersection notation;
- Find a missing probability from a list or two-way table, including algebraic terms;
- Understand conditional probabilities and decide if two events are independent;
- Draw a probability tree diagram based on given information, and use this to find probability and expected number of outcome;
- Understand selection with or without replacement;
- Calculate the probability of independent and dependent combined events;
- Use a two-way table to calculate conditional probability;
- Use a tree diagram to calculate conditional probability;
- Use a Venn diagram to calculate conditional probability;
- Compare experimental data and theoretical probabilities;
- Compare relative frequencies from samples of different sizes.

POSSIBLE SUCCESS CRITERIA

If the probability of outcomes are x, $2x$, $4x$, $3x$, calculate x.
Draw a Venn diagram of students studying French, German or both, and then calculate the probability that a student studies French given that they also study German.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Students should be given the opportunity to justify the probability of events happening or not happening in real-life and abstract contexts.

COMMON MISCONCEPTIONS

Probability without replacement is best illustrated visually and by initially working out probability ‘with’ replacement.
Not using fractions or decimals when working with probability trees.

NOTES

Encourage students to work ‘across’ the branches, working out the probability of each successive event. The probability of the combinations of outcomes should = 1.
Use problems involving ratio and percentage, similar to:

- A bag contains balls in the ratio 2 : 3 : 4. A ball is taken at random. Work out the probability that the ball will be … ;
- In a group of students 55% are boys, 65% prefer to watch film A, 10% are girls who prefer to watch film B. One student picked at random. Find the probability that this is a boy who prefers to watch film A (P6).

Emphasise that, were an experiment repeated, it will usually lead to different outcomes, and that increasing sample size generally leads to better estimates of probability and population characteristics.

UNIT 11: Multiplicative reasoning: direct and inverse proportion, relating to graph form for direct, compound measures, repeated proportional change	**Teaching Time** 6-8 hours

Return to Overview

SPECIFICATION REFERENCES

N3 recognise and use relationships between operations, including inverse operations (e.g. cancellation to simplify calculations and expressions); …

N12 interpret fractions and percentages as operators

N13 use standard units of mass, length, time, money and other measures (including standard compound measures) using decimal quantities where appropriate

R1 change freely between related standard units (e.g. time, length, area, volume/capacity, mass) and compound units (e.g. speed, rates of pay, prices, density, pressure) in numerical and algebraic contexts

R6 express a multiplicative relationship between two quantities as a ratio or a fraction

R8 relate ratios to fractions and to linear functions

R10 solve problems involving direct and inverse proportion, including graphical and algebraic representations

R11 use compound units such as speed, rates of pay, unit pricing, density and pressure

R13 understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; …

R14 … recognise and interpret graphs that illustrate direct and inverse proportion

R16 set up, solve and interpret the answers in growth and decay problems, including compound interest **and work with general iterative processes**

PRIOR KNOWLEDGE

Students should be able to find a percentage of an amount and relate percentages to decimals.
Students should be able to rearrange equations and use these to solve problems.
Knowledge of speed = distance/time, density = mass/volume.

KEYWORDS

Ration, proportion, best value, unitary, proportional change, compound measure, density, mass, volume, speed, distance, time, density, mass, volume, pressure, acceleration, velocity, inverse, direct, constant of proportionality

OBJECTIVES

By the end of the unit, students should be able to:

- Express a multiplicative relationship between two quantities as a ratio or a fraction, e.g. when A:B are in the ratio 3:5, A is $\frac{3}{5}B$. When $4a = 7b$, then $a = \frac{7b}{4}$ or a:b is 7:4;
- Solve proportion problems using the unitary method;
- Work out which product offers best value and consider rates of pay;
- Work out the multiplier for repeated proportional change as a single decimal number;
- Represent repeated proportional change using a multiplier raised to a power, use this to solve problems involving compound interest and depreciation;
- Understand and use compound measures and:
  - convert between metric speed measures;
  - convert between density measures;
  - convert between pressure measures;

- Use kinematics formulae from the formulae sheet to calculate speed, acceleration, etc (with variables defined in the question);
- Calculate an unknown quantity from quantities that vary in direct or inverse proportion;
- Recognise when values are in direct proportion by reference to the graph form, and use a graph to find the value of k in $y = kx$;
- Set up and use equations to solve word and other problems involving direct proportion (this is covered in more detail in unit 19);
- Relate algebraic solutions to graphical representation of the equations;
- Recognise when values are in inverse proportion by reference to the graph form;
- Set up and use equations to solve word and other problems involving inverse proportion, and relate algebraic solutions to graphical representation of the equations.

POSSIBLE SUCCESS CRITERIA

Change g/cm^3 to kg/m^3, kg/m^2 to g/cm^2, m/s to km/h.
Solve word problems involving direct and inverse proportion.
Understand direct proportion as: as x increases, y increases.
Understand inverse proportion as: as x increases, y decreases.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Speed/distance type problems that involve students justifying their reasons why one vehicle is faster than another.
Calculations involving value for money are a good reasoning opportunity that utilise different skills.
Working out best value of items using different currencies given an exchange rate.

NOTES

Include fractional percentages of amounts with compound interest and encourage use of single multipliers.
Amounts of money should be rounded to the nearest penny, but emphasise the importance of not rounding until the end of the calculation if doing in stages.
Use a formula triangle to help students see the relationship for compound measures – this will help them evaluate which inverse operations to use.
Help students to recognise the problem they are trying to solve by the unit measurement given, e.g. km/h is a unit of speed as it is speed divided by a time.
Kinematics formulae involve a constant acceleration (which could be zero).
Encourage students to write down the initial equation of proportionality and, if asked to find a formal relating two quantities, the constant of proportionality must be found.

UNIT 12: Similarity and congruence in 2D and 3D	Teaching Time 5-7 hours

Back to Overview

SPECIFICATION REFERENCES

R6 express a multiplicative relationship between two quantities as a ratio or a fraction
R12 compare lengths, areas and volumes using ratio notation; make links to similarity (including trigonometric ratios) and scale factors
G5 use the basic congruence criteria for triangles (SSS, SAS, ASA, RHS)
G6 apply angle facts, triangle congruence, similarity and properties of quadrilaterals to conjecture and derive results about angles and sides, including … the fact that the base angles of an isosceles triangle are equal, and use known results to obtain simple proofs
G17 … calculate: surface area and volume of spheres, pyramids, cones and composite solids
G19 apply the concepts of congruence and similarity, including the relationships between lengths, **areas and volumes** in similar figures

PRIOR KNOWLEDGE

Students should be able to recognise and enlarge shapes and calculate scale factors.
Students should have knowledge of how to calculate area and volume in various metric measures.
Students should be able to measure lines and angles, and use compasses, ruler and protractor to construct standard constructions.

KEYWORDS

Congruence, side, angle, compass, construction, shape, volume, length, area, volume, scale factor, enlargement, similar, perimeter, frustum

OBJECTIVES

By the end of the unit, students should be able to:

- Understand and use SSS, SAS, ASA and RHS conditions to prove the congruence of triangles using formal arguments, and to verify standard ruler and pair of compasses constructions;
- Solve angle problems by first proving congruence;
- Understand similarity of triangles and of other plane shapes, and use this to make geometric inferences;
- Prove that two shapes are similar by showing that all corresponding angles are equal in size and/or lengths of sides are in the same ratio/one is an enlargement of the other, giving the scale factor;
- Use formal geometric proof for the similarity of two given triangles;
- Understand the effect of enlargement on angles, perimeter, area and volume of shapes and solids;
- Identify the scale factor of an enlargement of a similar shape as the ratio of the lengths of two corresponding sides, using integer or fraction scale factors;
- Write the lengths, areas and volumes of two shapes as ratios in their simplest form;
- Find missing lengths, areas and volumes in similar 3D solids;
- Know the relationships between linear, area and volume scale factors of mathematically similar shapes and solids;

- Use the relationship between enlargement and areas and volumes of simple shapes and solids;
- Solve problems involving frustums of cones where you have to find missing lengths first using similar triangles.

POSSIBLE SUCCESS CRITERIA

Recognise that all corresponding angles in similar shapes are equal in size when the corresponding lengths of sides are not.
Understand that enlargement does not have the same effect on area and volume.
Understand, from the experience of constructing them, that triangles satisfying SSS, SAS, ASA and RHS are unique, but SSA triangles are not.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Multi-step questions which require calculating missing lengths of similar shapes prior to calculating area of the shape, or using this information in trigonometry or Pythagoras problems.

COMMON MISCONCEPTIONS

Students commonly use the same scale factor for length, area and volume.

NOTES

Encourage students to model consider what happens to the area when a 1 cm square is enlarged by a scale factor of 3.
Ensure that examples involving given volumes are used, requiring the cube root being calculated to find the length scale factor.
Make links between similarity and trigonometric ratios.

UNIT 13: Sine and cosine rules, $\frac{1}{2}ab \sin C$, trigonometry and Pythagoras' Theorem in 3D, trigonometric graphs, and accuracy and bounds

Return to Overview

SPECIFICATION REFERENCES

N16 apply and interpret limits of accuracy, **including upper and lower bounds**

A5 understand and use standard mathematical formulae; rearrange formulae to change the subject

A8 work with coordinates in all four quadrants

A12 recognise, sketch and interpret graphs of linear functions, quadratic functions, simple cubic functions, the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, **exponential, functions $y = k^x$ for positive values of k, and the trigonometric functions (with arguments in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size**

A13 **sketch translations and reflections of a given function**

G11 solve geometrical problems on coordinate axes

G20 know the formulae for: Pythagoras' Theorem $a^2 + b^2 = c^2$ and the trigonometric ratios, sine, cosine and tan; apply them to find angles and lengths in right-angled triangles **and, where possible, general triangles** in two **and three** dimensional figures

G21 know the exact values of sin θ and cos θ for θ = 0°, 30°, 45° , 60° and 90°; know the exact value of tan θ for θ = 0°, 30°, 45° and 60°

G22 **know and apply the sine rule $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$, and cosine rule $a^2 = b^2 + c^2 - 2bc \cos A$, to find unknown lengths and angles**

G23 **know and apply Area = $\frac{1}{2}ab \sin C$ to calculate the area, sides or angles of any triangle**

PRIOR KNOWLEDGE

Students should be able to use axes and coordinates to specify points in all four quadrants.
Students should be able to recall and apply Pythagoras' Theorem and trigonometric ratios.
Students should be able to substitute into formulae.

KEYWORDS

Axes, coordinates, sine, cosine, tan, angle, graph, transformations, side, angle, inverse, square root, 2D, 3D, diagonal, plane, cuboid

13a. Graphs of trigonometric functions (A8, A12, A13, G21)	Teaching time 5–7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise, sketch and interpret graphs of the trigonometric functions (in degrees) $y = \sin x$, $y = \cos x$ and $y = \tan x$ for angles of any size.
- Know the exact values of $\sin \theta$ and $\cos \theta$ for θ = 0°, 30°, 45° , 60° and 90° and exact value of $\tan \theta$ for θ = 0°, 30°, 45° and 60° and find them from graphs.
- Apply to the graph of $y = \mathrm{f}(x)$ the transformations $y = -\mathrm{f}(x)$, $y = \mathrm{f}(-x)$ for sine, cosine and tan functions $\mathrm{f}(x)$.
- Apply to the graph of $y = \mathrm{f}(x)$ the transformations $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$ for sine, cosine and tan functions $\mathrm{f}(x)$.

POSSIBLE SUCCESS CRITERIA

Match the characteristic shape of the graphs to their functions and transformations.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Match a given list of events/processes with their graph.
Calculate and justify specific coordinates on a transformation of a trigonometric function.

NOTES

Translations and reflections of functions are included in this specification, but not rotations or stretches.
This work could be supported by the used of graphical calculators or suitable ICT.
Students need to recall the above exact values for sin, cos and tan.

13b. Further trigonometry (N16, A5, A8, G11, G20, G22, G23)	Teaching time 8-10 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Know and apply Area = $\frac{1}{2}ab$ sin C to calculate the area, sides or angles of any triangle.
- Know the sine and cosine rules, and use to solve 2D problems (including involving bearings).
- Use the sine and cosine rules to solve 3D problems.
- Understand the language of planes, and recognise the diagonals of a cuboid.
- Solve geometrical problems on coordinate axes.
- Understand, recall and use trigonometric relationships and Pythagoras' Theorem in right-angled triangles, and use these to solve problems in 3D configurations.
- Calculate the length of a diagonal of a cuboid.
- Find the angle between a line and a plane.

POSSIBLE SUCCESS CRITERIA

Find the area of a segment of a circle given the radius and length of the chord.
Justify when to use the cosine rule, sine rule, Pythagoras' Theorem or normal trigonometric ratios to solve problems.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Triangles formed in a semi-circle can provide links with other areas of mathematics.

COMMON MISCONCEPTIONS

Not using the correct rule, or attempting to use 'normal trig' in non-right-angled triangles.
When finding angles students will be unable to rearrange the cosine rule or fail to find the inverse of cos θ.

NOTES

The cosine rule is used when we have SAS and used to find the side opposite the 'included' angle or when we have SSS to find an angle.
Ensure that finding angles with 'normal trig' is refreshed prior to this topic.
Students may find it useful to be reminded of simple geometrical facts, i.e. the shortest side is always opposite the shortest angle in a triangle.
The sine and cosine rules and general formula for the area of a triangle are not given on the formulae sheet.
In multi-step questions emphasise the importance of not rounding prematurely and using exact values where appropriate.
Whilst 3D coordinates are not included in the programme of study, they provide a visual introduction to trigonometry in 3D.

UNIT 14: Statistics and sampling, cumulative frequency and histograms

Return to Overview

SPECIFICATION REFERENCES

S1 infer properties of populations or distributions from a sample, whilst knowing the limitations of sampling apply statistics to describe a population

S3 **interpret and construct diagrams for grouped discrete data and continuous data, i.e. histograms with equal and unequal class intervals and cumulative frequency graphs, and know their appropriate use**

S4 interpret, analyse and compare the distributions of data sets from univariate empirical distributions through:

- Appropriate graphical representation involving discrete, continuous and grouped data, **including box plots**
- appropriate measures of central tendency (median, mean, mode and modal class) and spread (range, including consideration of outliers, **quartiles and inter-quartile range)**

S5 apply statistics to describe a population

PRIOR KNOWLEDGE

Students should understand the different types of data: discrete/continuous.
Students should have experience of inequality notation.
Students should be able to multiply a fraction by a number.
Students should understand the data handling cycle.

KEYWORDS

Sample, population, fraction, decimal, percentage, bias, stratified sample, random, cumulative frequency, box plot, histogram, frequency density, frequency, mean, median, mode, range, lower quartile, upper quartile, interquartile range, spread, comparison, outlier

14a. Collecting data (S1)	**Teaching time** 5–7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Specify the problem and plan:
    - decide what data to collect and what analysis is needed;
    - understand primary and secondary data sources;
    - consider fairness;
- Understand what is meant by a sample and a population;
- Understand how different sample sizes may affect the reliability of conclusions drawn;
- Identify possible sources of bias and plan to minimise it;
- Write questions to eliminate bias, and understand how the timing and location of a survey can ensure a sample is representative (see note);

POSSIBLE SUCCESS CRITERIA

Explain why a sample may not be representative of a whole population.
Carry out their own statistical investigation and justify how sources of bias have been eliminated.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

When using a sample of a population to solve contextual problem, students should be able to justify why the sample may not be representative the whole population.

NOTES

Emphasise the difference between primary and secondary sources and remind students about the difference between discrete and continuous data.
Discuss sample size and mention that a census is the whole population (the UK census takes place every 10 years in a year ending with a 1 – the next one is due in 2021).
Specifying the problem and planning for data collection is not included in the programme of study, but is a prerequisite to understanding the context of the topic.
Writing a questionnaire is also not included in the programme of study, but remains a good topic for demonstrating bias and ways to reduce bias in terms of timing, location and question types.

14b. Cumulative frequency, box plots and histograms (S3, S4, S5)

Teaching time 5-7 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Use statistics found in all graphs/charts in this unit to describe a population;
- Know the appropriate uses of cumulative frequency diagrams;
- Construct and interpret cumulative frequency tables, cumulative frequency graphs/diagrams and from the graph:
  - estimate frequency greater/less than a given value;
  - find the median and quartile values and interquartile range;
- Compare the mean and range of two distributions, or median and interquartile range, as appropriate;
- Interpret box plots to find median, quartiles, range and interquartile range and draw conclusions;
- Produce box plots from raw data and when given quartiles, median and identify any outliers;
- Know the appropriate uses of histograms;
- Construct and interpret histograms from class intervals with unequal width;
- Use and understand frequency density;
- From histograms:
  - complete a grouped frequency table;
  - understand and define frequency density;
- Estimate the mean and median from a histogram with unequal class widths or any other information from a histogram, such as the number of people in a given interval.

POSSIBLE SUCCESS CRITERIA

Construct cumulative frequency graphs, box plots and histograms from frequency tables.
Compare two data sets and justify their comparisons based on measures extracted from their diagrams where appropriate in terms of the context of the data.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Interpret two or more data sets from box plots and relate the key measures in the context of the data.
Given the size of a sample and its box plot calculate the proportion above/below a specified value.

COMMON MISCONCEPTIONS

Labelling axes incorrectly in terms of the scales, and also using 'Frequency' instead of 'Frequency Density' or 'Cumulative Frequency'.
Students often confuse the methods involved with cumulative frequency, estimating the mean and histograms when dealing with data tables.

NOTES

Ensure that axes are clearly labelled.

As a way to introduce measures of spread, it may be useful to find mode, median, range and interquartile range from stem and leaf diagrams (including back-to-back) to compare two data sets.

As an extension, use the formula for identifying an outlier, (i.e. if data point is below LQ – 1.5 × IQR or above UQ + 1.5 × IQR, it is an outlier). Get them to identify outliers in the data, and give bounds for data.

UNIT 15: Quadratics, expanding more than two brackets, sketching graphs, graphs of circles, cubes and quadratics

Teaching time
6-8 hours

Return to Overview

SPECIFICATION REFERENCES

N8 Calculate exactly with … **surds** …
A4 simplify and manipulate algebraic expressions … by: expanding products of two **or more** binomials
A11 identify and interpret roots, intercepts, turning points of quadratic functions graphically; … **identify turning points by completing the square**
A12 recognise, sketch and interpret graphs of … quadratic functions, simple cubic functions …
A18 solve quadratic equations (**including those that require rearrangement)** …; find approximate solutions using a graph
A19 solve two simultaneous equations in two variables (linear/linear **or linear/quadratic**) algebraically; find approximate solutions using a graph
A20 **find approximate solutions to equations numerically using iteration**
A21 … derive an equation (or two simultaneous equations), solve the equation(s) and interpret the solution.
A22 solve linear inequalities in one or **two** variable**(s), and quadratic inequalities in one variable**; represent the solution set on a number line, **using set notation and on a graph**
R16 … **work with general iterative processes**

PRIOR KNOWLEDGE

Students should be able to solve quadratics and linear equations.
Students should be able to solve simultaneous equations algebraically.

KEYWORDS

Sketch, estimate, quadratic, cubic, function, factorising, simultaneous equation, graphical, algebraic

OBJECTIVES

By the end of the unit, students should be able to:

- Sketch a graph of a quadratic function, by factorising or by using the formula, identifying roots, y-intercept and turning point by completing the square;
- Be able to identify from a graph if a quadratic equation has any real roots;
- Find approximate solutions to quadratic equations using a graph;
- Expand the product of more than two linear expressions;
- Sketch a graph of a quadratic function and a linear function, identifying intersection points;
- Sketch graphs of simple cubic functions, given as three linear expressions;
- Solve simultaneous equations graphically:
    - find approximate solutions to simultaneous equations formed from one linear function and one quadratic function using a graphical approach;
    - find graphically the intersection points of a given straight line with a circle;
    - solve simultaneous equations representing a real-life situation graphically, and interpret the solution in the context of the problem;
- Solve quadratic inequalities in one variable, by factorising and sketching the graph to find critical values;

- Represent the solution set for inequalities using set notation, i.e. curly brackets and 'is an element of' notation;
  - for problems identifying the solutions to two different inequalities, show this as the intersection of the two solution sets, i.e. solution of $x^2 - 3x - 10 < 0$ as $\{x: -3 < x < 5\}$;
- Solve linear inequalities in two variables graphically;
- Show the solution set of several inequalities in two variables on a graph;
- Use iteration with simple converging sequences.

POSSIBLE SUCCESS CRITERIA

Expand $x(x - 1)(x + 2)$.
Expand $(x - 1)^3$.
Expand $(x + 1)(x + 2)(x - 1)$.
Sketch $y = (x + 1)^2(x - 2)$.
Interpret a pair of simultaneous equations as a pair of straight lines and their solution as the point of intersection.
Be able to state the solution set of $x^2 - 3x - 10 < 0$ as $\{x: x < -3\} \cup \{x: x > 5\}$.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Match equations to their graphs and to real-life scenarios.
"Show that"-type questions will allow students to show a logical and clear chain of reasoning.

COMMON MISCONCEPTIONS

When estimating values from a graph, it is important that students understand it is an 'estimate'.
It is important to stress that when expanding quadratics, the x terms are also collected together.
Quadratics involving negatives sometimes cause numerical errors.

NOTES

The extent of algebraic iteration required needs to be confirmed.
You may want to extend the students to include expansions of more than three linear expressions.
Practise expanding 'double brackets' with all combinations of positives and negatives.
Set notation is a new topic.

UNIT 16: Circle theorems and circle geometry

Return to Overview

SPECIFICATION REFERENCES

A16 **recognise and use the equation of a circle with centre at the origin; find the equation of a tangent to a circle at a given point**

G9 identify and apply circle definitions and properties, including: centre, radius, chord, diameter, circumference, tangent, arc, sector and segment

G10 **apply and prove the standard circle theorems concerning angles, radii, tangents and chords, and use them to prove related results**

PRIOR KNOWLEDGE

Students should have practical experience of drawing circles with compasses.
Students should recall the words, centre, radius, diameter and circumference.
Students should recall the relationship of the gradient between two perpendicular lines.
Students should be able to find the equation of the straight line, given a gradient and a coordinate.

KEYWORDS

Radius, centre, tangent, circumference, diameter, gradient, perpendicular, reciprocal, coordinate, equation, substitution, chord, triangle, isosceles, angles, degrees, cyclic quadrilateral, alternate, segment, semicircle, arc, theorem

16a. Circle theorems (G9, G10)	**Teaching time** 4-6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recall the definition of a circle and identify (name) and draw parts of a circle, including sector, tangent, chord, segment;
- Prove and use the facts that:
  - the angle subtended by an arc at the centre of a circle is twice the angle subtended at any point on the circumference;
  - the angle in a semicircle is a right angle;
  - the perpendicular from the centre of a circle to a chord bisects the chord;
  - angles in the same segment are equal;
  - alternate segment theorem;
  - opposite angles of a cyclic quadrilateral sum to 180°;
- Understand and use the fact that the tangent at any point on a circle is perpendicular to the radius at that point;
- Find and give reasons for missing angles on diagrams using:
  - circle theorems;
  - isosceles triangles (radius properties) in circles;
  - the fact that the angle between a tangent and radius is 90°;
  - the fact that tangents from an external point are equal in length.

POSSIBLE SUCCESS CRITERIA

Justify clearly missing angles on diagrams using the various circle theorems.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Problems that involve a clear chain of reasoning and provide counter-arguments to statements. Can be linked to other areas of mathematics by incorporating trigonometry and Pythagoras' Theorem.

COMMON MISCONCEPTIONS

Much of the confusion arises from mixing up the diameter and the radius.

NOTES

Reasoning needs to be carefully constructed and correct notation should be used throughout.
Students should label any diagrams clearly, as this will assist them; particular emphasis should be made on labelling any radii in the first instance.

16b. Circle geometry	**Teaching time**
(A16)	4-6 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Select and apply construction techniques and understanding of loci to draw graphs based on circles and perpendiculars of lines;
- Find the equation of a tangent to a circle at a given point, by:
  - finding the gradient of the radius that meets the circle at that point (circles all centre the origin);
  - finding the gradient of the tangent perpendicular to it;
  - using the given point;
- Recognise and construct the graph of a circle using $x^2 + y^2 = r^2$ for radius r centred at the origin of coordinates.

POSSIBLE SUCCESS CRITERIA

Find the gradient of a radius of a circle drawn on a coordinate grid and relate this to the gradient of the tangent.
Justify the relationship between the gradient of a tangent and the radius.
Produce an equation of a line given a gradient and a coordinate.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Justify if a straight-line graph would pass through a circle drawn on a coordinate grid.

COMMON MISCONCEPTIONS

Students find it difficult working with negative reciprocals of fractions and negative fractions.

NOTES

Work with positive gradients of radii initially and review reciprocals prior to starting this topic.
It is useful to start this topic through visual proofs, working out the gradient of the radius and the tangent, before discussing the relationship.

UNIT 17: Changing the subject of formulae (more complex), algebraic fractions, solving equations arising from algebraic fractions, rationalising surds, proof | **Teaching time** 6-8 hours

Return to Overview

SPECIFICATION REFERENCES

N8 … **simplify surd expressions involving squares (e.g. √12 = √(4 × 3) = √4 × √3 = 2√3) and rationalise denominators**

A4 simplify and manipulate algebraic expressions (including those involving surds **and algebraic fractions**) by:

- collecting like terms
- multiplying a single term over a bracket
- taking out common factors
- expanding products of two **or more** binomials
- factorising quadratic expressions of the form $x^2 + bx + c$, including the difference of two squares; **factorising quadratic expressions of the form $ax^2 + bx + c$**
- simplifying expressions involving sums, products and powers, including the laws of indices

A5 … rearrange formulae to change the subject

A6 … argue mathematically to show algebraic expressions are equivalent, and use algebra to support and construct arguments **and proofs**

A7 where appropriate, interpret simple expressions as functions with inputs and outputs; **interpret the reverse process as the 'inverse function'; interpret the succession of two functions as a 'composite function' (the use of formal function notation is expected)**

A18 solve quadratic equations **(including those that require rearrangement)** algebraically by factorising, …

PRIOR KNOWLEDGE

Students should be able to simplify surds.
Students should be able to use negative numbers with all four operations.
Students should be able to recall and use the hierarchy of operations.

KEYWORDS

Rationalise, denominator, surd, rational, irrational, fraction, equation, rearrange, subject, proof, function notation, inverse, evaluate

OBJECTIVES

By the end of the unit, students should be able to:

- Rationalise the denominator involving surds;
- Simplify algebraic fractions;
- Multiply and divide algebraic fractions;
- Solve quadratic equations arising from algebraic fraction equations;
- Change the subject of a formula, including cases where the subject occurs on both sides of the formula, or where a power of the subject appears;
- Change the subject of a formula such as $\frac{1}{f} = \frac{1}{u} + \frac{1}{v}$, where all variables are in the denominators;

- Solve 'Show that' and proof questions using consecutive integers (n, n + 1), squares a^2, b^2, even numbers $2n$, odd numbers $2n$ +1;
- Use function notation;
- Find f(x) + g(x) and f(x) – g(x), 2f(x), f(3x) etc algebraically;
- Find the inverse of a linear function;
- Know that $f^{-1}(x)$ refers to the inverse function;
- For two functions f(x) and g(x), find gf(x).

POSSIBLE SUCCESS CRITERIA

Rationalise: $\frac{1}{\sqrt{3}-1}$, $\frac{1}{\sqrt{3}}$, ($\sqrt{18}$ + 10) +$\sqrt{2}$.
Explain the difference between rational and irrational numbers.
Given a function, evaluate f(2).
When g(x) = 3 – 2x, find g^{-1} (x).

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Formal proof is an ideal opportunity for students to provide a clear logical chain of reasoning providing links with other areas of mathematics.

COMMON MISCONCEPTIONS

$\sqrt{3}$ x $\sqrt{3}$ = 9 is often seen.
When simplifying involving factors, students often use the 'first' factor that they find and not the LCM.

NOTES

It is useful to generalise $\sqrt{m} \times \sqrt{m} = m$.
Revise the difference of two squares to show why we use, for example, ($\sqrt{3}$ – 2) as the multiplier to rationalise ($\sqrt{3}$ + 2).
Link collecting like terms to simplifying surds (Core 1 textbooks are a good source for additional work in relation to simplifying surds).
Practice factorisation where the factor may involve more than one variable.
Emphasise that, by using the LCM for the denominator, the algebraic manipulation is easier.

UNIT 18: Vectors and geometric proof

Teaching time
8-10 hours

Return to Overview

SPECIFICATION REFERENCES

G25 apply addition and subtraction of vectors, multiplication of vectors by a scalar, and diagrammatic and column representations of vectors; **use vectors to construct geometric arguments and proof**

PRIOR KNOWLEDGE

Students will have used vectors to describe translations and will have knowledge of Pythagoras' Theorem and the properties of triangles and quadrilaterals.

KEYWORDS

Vector, direction, magnitude, scalar, multiple, parallel, collinear, proof, ratio, column vector

OBJECTIVES

By the end of the unit, students should be able to:

- Understand and use vector notation, including column notation, and understand and interpret vectors as displacement in the plane with an associated direction.
- Understand that 2**a** is parallel to **a** and twice its length, and that **a** is parallel to –**a** in the opposite direction.
- Represent vectors, combinations of vectors and scalar multiples in the plane pictorially.
- Calculate the sum of two vectors, the difference of two vectors and a scalar multiple of a vector using column vectors (including algebraic terms).
- Find the length of a vector using Pythagoras' Theorem.
- Calculate the resultant of two vectors.
- Solve geometric problems in 2D where vectors are divided in a given ratio.
- Produce geometrical proofs to prove points are collinear and vectors/lines are parallel.

POSSIBLE SUCCESS CRITERIA

Add and subtract vectors algebraically and use column vectors.
Solve geometric problems and produce proofs.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

"Show that"-type questions are an ideal opportunity for students to provide a clear logical chain of reasoning providing links with other areas of mathematics, in particular algebra.
Find the area of a parallelogram defined by given vectors.

COMMON MISCONCEPTIONS

Students find it difficult to understand that parallel vectors are equal as they are in different locations in the plane.

NOTES

Students find manipulation of column vectors relatively easy compared to pictorial and algebraic manipulation methods – encourage them to draw any vectors they calculate on the picture.
Geometry of a hexagon provides a good source of parallel, reverse and multiples of vectors.
Remind students to underline vectors or use an arrow above them, or they will be regarded as just lengths.
Extend geometric proofs by showing that the medians of a triangle intersect at a single point.
3D vectors or **i**, **j** and **k** notation can be introduced and further extension work can be found in GCE Mechanics 1 textbooks.

UNIT 19: Direct and indirect proportion: using statements of proportionality, reciprocal and exponential graphs, rates of change in graphs, functions, transformations of graphs

Return to Overview

SPECIFICATION REFERENCES

A7 where appropriate, interpret simple expressions as functions with inputs and outputs; …

A12 recognise, sketch and interpret graphs of the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$, **exponential functions $y = k^x$ for positive values of k** …

A13 **sketch translations and reflections of a given function**

A14 plot and interpret reciprocal graphs and **exponential graphs** …

A15 **calculate or estimate gradients of graphs and areas under graphs (including quadratic and other non-linear graphs) and interpret results in cases such distance–time graphs, velocity–time graphs and graphs in financial contexts (this does not include calculus)**

A21 translate simple situations or procedures into algebraic expressions or formulae; …

R7 understand and use proportion as equality of ratios

R10 solve problems involving direct and inverse proportion, including graphical and algebraic representations

R11 use compound units such as speed, rates of pay, unit pricing, density and pressure

R13 understand that X is inversely proportional to Y is equivalent to X is proportional to $\frac{1}{Y}$; **construct and** interpret equations that describe direct and inverse proportion

R14 interpret the gradient of a straight line graph as a rate of change; recognise and interpret graphs that illustrate direct and inverse proportion

R15 **interpret the gradient at a point on a curve as the instantaneous rate of change; apply the concepts of average and instantaneous rate of change (gradients of chords and tangents) in numerical, algebraic and graphical contexts (this does not include calculus**

R16 set up, solve and interpret the answers in growth and decay problems …

PRIOR KNOWLEDGE

Students should be able to draw linear and quadratic graphs.
Students should be able to calculate the gradient of a linear function between two points.
Students should recall transformations of trigonometric functions.
Students should have knowledge of writing statements of direct proportion and forming an equation to find values.

KEYWORDS

Reciprocal, linear, gradient, quadratic, exponential, functions, direct, indirect, proportion, estimate, area, rate of change, distance, time, velocity, transformations, cubic, transformation, constant of proportionality

19a. Reciprocal and exponential graphs; Gradient and area under graphs
(R11, R14, R15, R16, A7, A12, A13, A14, A15)

Teaching time
6-8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise, sketch and interpret graphs of the reciprocal function $y = \frac{1}{x}$ with $x \neq 0$
- State the value of x for which the equation is not defined;
- Recognise, sketch and interpret graphs of exponential functions $y = k^x$ for positive values of k and integer values of x;
- Use calculators to explore exponential growth and decay;
- Set up, solve and interpret the answers in growth and decay problems;
- Interpret and analyse transformations of graphs of functions and write the functions algebraically, e.g. write the equation of $\mathrm{f}(x) + a$, or $\mathrm{f}(x - a)$:
  - apply to the graph of $y = \mathrm{f}(x)$ the transformations $y = -\mathrm{f}(x)$, $y = \mathrm{f}(-x)$ for linear, quadratic, cubic functions;
  - apply to the graph of y = $\mathrm{f}(x)$ the transformations $y = \mathrm{f}(x) + a$, $y = \mathrm{f}(x + a)$ for linear, quadratic, cubic functions;
- Estimate area under a quadratic or other graph by dividing it into trapezia;
- Interpret the gradient of linear or non-linear graphs, and estimate the gradient of a quadratic or non-linear graph at a given point by sketching the tangent and finding its gradient;
- Interpret the gradient of non-linear graph in curved distance–time and velocity–time graphs:
  - for a non-linear distance–time graph, estimate the speed at one point in time, from the tangent, and the average speed over several seconds by finding the gradient of the chord;
  - for a non-linear velocity–time graph, estimate the acceleration at one point in time, from the tangent, and the average acceleration over several seconds by finding the gradient of the chord;
- Interpret the gradient of a linear or non-linear graph in financial contexts;
- Interpret the area under a linear or non-linear graph in real-life contexts;
- Interpret the rate of change of graphs of containers filling and emptying;
- Interpret the rate of change of unit price in price graphs.

POSSIBLE SUCCESS CRITERIA

Explain why you cannot find the area under a reciprocal or tan graph.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Interpreting many of these graphs in relation to their specific contexts.

COMMON MISCONCEPTIONS

The effects of transforming functions is often confused.

NOTES

Formal function notation along with inverse and composite functions will have been encountered but are topics that students may need to be reminded about.
Translations and reflections of functions are included in this specification, but not rotations or stretches.
Financial contexts could include percentage or growth rate.
When interpreting rates of change with graphs of containers filling and emptying, a steeper gradient means a faster rate of change.
When interpreting rates of change of unit price in price graphs, a steeper graph means larger unit price.

19b. Direct and inverse proportion (A21, R7, R10, R13, R14) — **Teaching time** 6-8 hours

OBJECTIVES

By the end of the sub-unit, students should be able to:

- Recognise and interpret graphs showing direct and inverse proportion;
- Identify direct proportion from a table of values, by comparing ratios of values, for x squared and x cubed relationships;
- Write statements of proportionality for quantities proportional to the square, cube or other power of another quantity;
- Set up and use equations to solve word and other problems involving direct proportion;
- Use $y = kx$ to solve direct proportion problems, including questions where students find k, and then use k to find another value;
- Solve problems involving inverse proportion using graphs by plotting and reading values from graphs;
- Solve problems involving inverse proportionality;
- Set up and use equations to solve word and other problems involving direct proportion or inverse proportion.

POSSIBLE SUCCESS CRITERIA

Understand that when two quantities are in direct proportion, the ratio between them remains constant.
Know the symbol for 'is proportional to'.

OPPORTUNITIES FOR REASONING/PROBLEM SOLVING

Justify and infer relationships in real-life scenarios to direct and inverse proportion such as ice cream sales and sunshine.

COMMON MISCONCEPTIONS

Direct and inverse proportion can get mixed up.

NOTES

Consider using science contexts for problems involving inverse proportionality, e.g. volume of gas inversely proportional to the pressure or frequency is inversely proportional to wavelength.